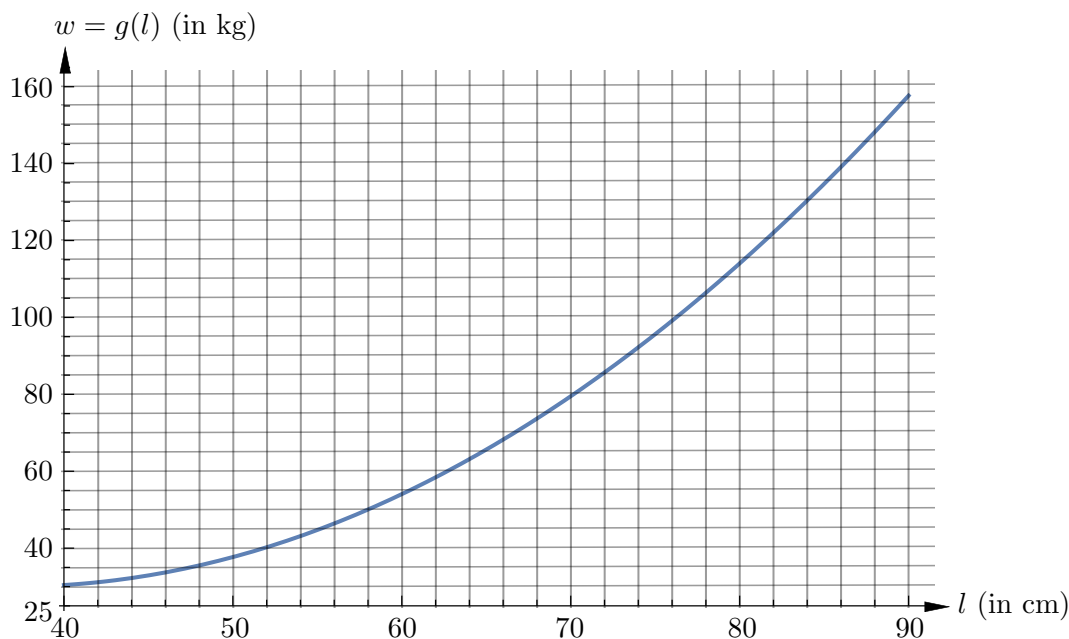


9. [12 points] You would like to investigate the relationship between the swimming speed S (in cm/sec), the weight w (in kg) and the length l (in cm) of salmon. Let f and g be invertible functions that take as input the length of the salmon and give as output its swimming speed and weight respectively. In other words, $S = f(l)$ and $w = g(l)$. You measured the swimming speed and the length of six salmons. The data you obtained is summarized in the table below.

	Salmon 1	Salmon 2	Salmon 3	Salmon 4	Salmon 5	Salmon 6
l	60	80	50	85	76	40
$S = f(l)$	148	161	140	163	158	130

The graph of g is drawn below.



- a. [6 points] Find the value of the following expressions. Include units.

$$g^{-1}(100) = \underline{\hspace{2cm}} \quad f(80) = \underline{\hspace{2cm}} \quad f^{-1}(140) = \underline{\hspace{2cm}}$$

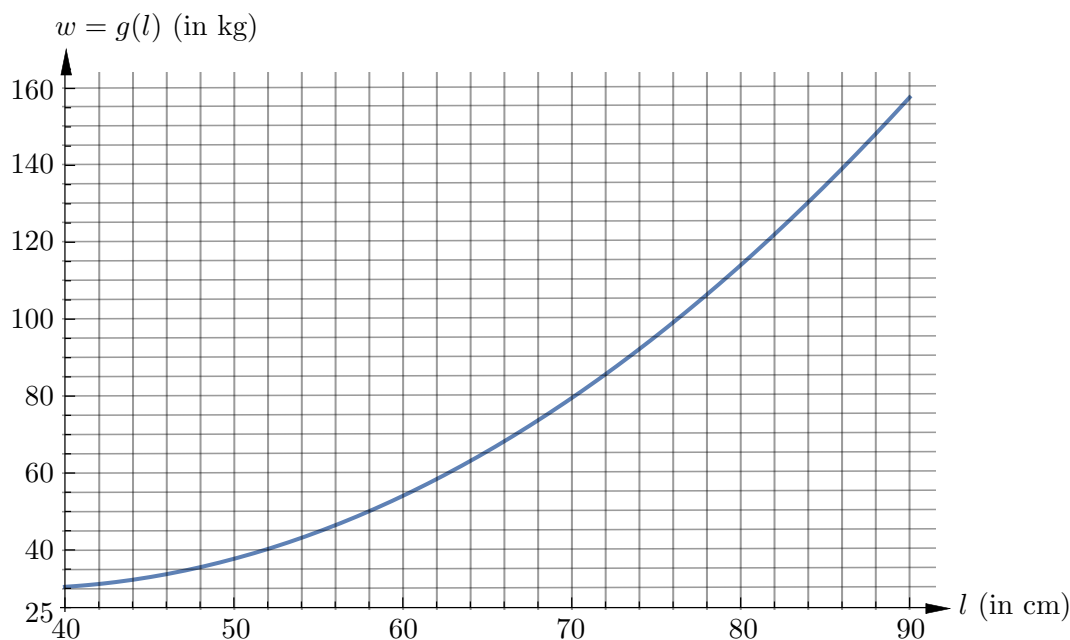
$$\boxed{\text{Solution: } g^{-1}(100) = 76 \text{ cm, } f(80) = 161 \text{ cm/s, } f^{-1}(140) = 50 \text{ cm.}}$$

- b. [2 points] What is the **weight** of a salmon that swims at a speed of 130 cm/sec?

$$\boxed{\text{Solution: A salmon that can swim at 130 cm/sec has length 40 cm. According to the graph, the weight of a salmon 40 cm long is 30 kg.}}$$

The graph and table from the previous page has been copied here for your convenience

	Salmon 1	Salmon 2	Salmon 3	Salmon 4	Salmon 5	Salmon 6
l	60	80	50	85	76	40
$S = f(l)$	148	161	140	163	158	130



- c. [4 points] Find the average rate of change of the **weight** of a salmon as a function of its swimming speed over the interval between $S = 148$ and $S = 158$. Show all your work to receive full credit. Include units.

Solution: When the swimming speed of a salmon is 148 cm per second, its length is 60 cm. If the swimming speed of a salmon is 158 cm per second, its length is 76 cm. The weight of a salmon of 60 cm and 76 cm in length is 55 kg and 100 kg. Hence the average rate of change of the weight of a salmon over the interval between $S = 148$ and $S = 158$ is given by

$$\frac{g(f^{-1}(158)) - g(f^{-1}(148))}{158 - 148} = \frac{100 - 55}{158 - 148} = 4.5 \frac{\text{kg}}{\text{cm/sec}}.$$