5. [12 points] Three researchers Dr. Banner, Dr. Storm and Dr. Kyle are studying a population of an alien species, commonly known as CATS (Category A Threats). The study of CATS started in 2010 and the scientists observed a population of 125 CATS in 2012 and a population of 600 CATS in 2014. In all the functions below, the variable t represents the number of *years after 2010*. Show all your work.

a. [3 points] Doctor Banner believes the CATS population is described by the linear function $R(t)$. Find a formula for $R(t)$.

Solution: Slope:

$$m = \frac{600 - 125}{4 - 2} = 237.5$$

Point-Slope formula:

$$B(t) = 125 + 237.5(t - 2)$$

$$B(t) = 237.5t - 350$$

b. [4 points] Doctor Storm thinks that a linear model is not adequate to describe the population of CATS. She believes that the number of CATS can be described by a quadratic function $S(t)$ whose minimum occurred in 2012. Find a formula for $S(t)$.

Solution: Vertex at $(2, 125)$, so

$$S(t) = a(t - 2)^2 + 125.$$

Plug in $(4, 600)$ to solve for a.

$$600 = a(4 - 2)^2 + 125$$

$$475 = 4a$$

$$a = 118.75$$

Hence $S(t) = 118.75(t - 2)^2 + 125$

c. [5 points] On the other hand, Doctor Kyle strongly believes that the CATS' population size must grow exponentially. He describes the population of CATS using the exponential function $K(t)$. Find a formula for $K(t)$. Your answer must be in **exact form**.

Solution: Find b. $600 = ab^4$ and $125 = ab^2$ yields $b^2 = 4.8$. Hence $b = \sqrt{4.8}$.
Plug in a point and solve for a.

$$125 = a(\sqrt{4.8})^2$$

$$125 = 4.8a, \qquad a = \frac{125}{4.8}$$

Hence,

$$K(t) = \frac{125}{4.8}(4.8)^{t/2}$$