8. [12 points] Let
\[ V(x) = -\frac{1}{2}x^2 + \frac{9}{2}x + \frac{47}{8} \quad \text{for} \quad 0 \leq x \leq 10 \]
be the number of viewers of a 10-minute interview (in millions), \( x \) minutes after the interview started.

a. [5 points] Write the quadratic function \( V(x) \) in vertex form by completing the square. Show all your work carefully, step by step to receive full credit.

\[
\begin{align*}
V(x) &= -\frac{1}{2}(x^2 - 9x) + \frac{47}{8} \\
V(x) &= -\frac{1}{2}\left(x^2 - 9x + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2\right) + \frac{47}{8} \\
V(x) &= -\frac{1}{2}\left(x - \frac{9}{2}\right)^2 - \frac{81}{4} + \frac{47}{8} \\
V(x) &= -\frac{1}{2}\left(x - \frac{9}{2}\right)^2 + \frac{81}{8} + \frac{47}{8} \\
V(x) &= -\frac{1}{2}\left(x - \frac{9}{2}\right)^2 + 16
\end{align*}
\]

b. [3 points] In how many minutes after the beginning of the interview did the number of viewers reach its minimum and maximum, respectively?

\[
\text{Solution:} \quad \text{The minimum occurs at one of the endpoints. One checks that it occurs at } x = 10. \quad \text{The maximum occurs at the } x\text{-coordinate of the vertex i.e., at } x = 4.5.
\]

c. [4 points] For how long will the number of viewers of the interview be more than 10 million? Recall that
\[ V(x) = -\frac{1}{2}x^2 + \frac{9}{2}x + \frac{47}{8} \quad \text{for} \quad 0 \leq x \leq 10. \]

Solve this problem algebraically. Your answer must be in exact form. Show all your work.

\[
\text{Solution:} \quad -0.5x^2 + 4.5x + \frac{47}{8} = 10 \quad \text{or} \quad -0.5x^2 + 4.5x - 4.125 = 0
\]

Using the quadratic formula we get two solutions in the function’s domain:
\[ x = 4.5 \pm \sqrt{(4.5)^2 - 4(-0.5)(-4.125)} = 4.5 \pm 2\sqrt{3}. \]

Hence the number of viewers is larger than 10 millions for \( 4\sqrt{3} \) minutes.