8. [12 points] Let

$$V(x) = -\frac{1}{2}x^2 + \frac{9}{2}x + \frac{47}{8} \qquad \text{for} \qquad 0 \le x \le 10$$

be the number of viewers of a 10-minute interview (in millions), x minutes after the interview started.

a. [5 points] Write the quadratic function $V(x)$ in vertex form by completing the square. Show all your work carefully, step by step to receive full credit.

Solution:

$$V(x) = -\frac{1}{2}(x^2 - 9x) + \frac{47}{8}$$
$$V(x) = -\frac{1}{2}\left(x^2 - 9x + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2\right) + \frac{47}{8}$$
$$V(x) = -\frac{1}{2}\left(\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right) + \frac{47}{8}$$
$$V(x) = -\frac{1}{2}\left(x - \frac{9}{2}\right)^2 + \frac{81}{8} + \frac{47}{8}$$
$$V(x) = -\frac{1}{2}\left(x - \frac{9}{2}\right)^2 + 16$$

b. [3 points] In how many minutes after the beginning of the interview did the number of viewers reach its minimum and maximum, respectively?

Solution: The minimum occurs at one of the endpoints. One checks that it occurs at $x = 10$. The maximum occurs at the x-coordinate of the vertex i.e., at $x = 4.5$.

c. [4 points] For how long will the number of viewers of the interview be more than 10 million? Recall that

$$V(x) = -\frac{1}{2}x^2 + \frac{9}{2}x + \frac{47}{8} \qquad \text{for} \qquad 0 \le x \le 10.$$

Solve this problem algebraically. Your answer must be in **exact form**. Show all your work.

Solution: $-0.5x^2 + 4.5x + \dfrac{47}{8} = 10$ or $-0.5x^2 + 4.5x - 4.125 = 0$
Using the quadratic formula we get two solutions in the function's domain:

$$x = 4.5 \pm \sqrt{(4.5)^2 - 4(-0.5)(-4.125)} = 4.5 \pm 2\sqrt{3}.$$

Hence the number of viewers is larger than 10 millions for $4\sqrt{3}$ minutes.