5. [11 points] In both parts of this problem, you should **show your work**, and make sure your answers are **exact** and written *in the spaces provided*.

a. [6 points] Kayla was cultivating a strain of bacteria in her lab, and noticed that the mass of her bacterial culture was growing exponentially. She started the experiment at 9 a.m. and ended it at 5 p.m., at which point she had 234 grams of bacteria. Find a formula expressing the mass of her culture $m(t)$ (in grams) as a function of the time t, measured in hours after 9 a.m., given that the mass of her culture was 20 grams at noon.

Solution: Since $m(t)$ is exponential, our formula is of the form $m(t) = ab^t$ for some constants a and b. We know that $m(3) = 20$ and $m(8) = 234$, which gives us:

$$ab^8 = 234$$
$$ab^3 = 20$$

and dividing the first equation by the second gives us:

$$b^5 = 11.7$$

so $b = 11.7^{\frac{1}{5}}$. To get a, we plug this back into the second equation, which gives us:

$$a\left(11.7^{\frac{1}{5}}\right)^3 = 20$$

and so $a = 20 \cdot 11.7^{-\frac{3}{5}}$.

$$m(t) = \underline{\quad 20 \cdot 11.7^{-\frac{3}{5}} \cdot (11.7^{\frac{1}{5}})^t \quad}$$

b. [5 points] A 10 liter bottle is filled completely with a combination of oil and vinegar. Each kilogram of oil takes up 1.25 liters, while each kilogram of vinegar takes up 1 liter. Let $N(\ell)$ be the amount of vinegar (measured in kilograms) in the bottle when it is filled with ℓ kilograms of oil. Find a formula for $N(\ell)$ in terms of ℓ and indicate the domain on which your formula is valid. *Note: there are practical considerations for your domain in this problem.*

Solution: Since the oil and vinegar fill up the bottle completely, we have $(1.25 \cdot \ell) + (1 \cdot N(\ell)) = 10$, and so $N(\ell) = 10 - 1.25 \cdot \ell$. Of course, ℓ cannot be negative, and the largest ℓ can be is when the bottle is filled with oil and has no vinegar. In other words, ℓ is the largest when $1.25\ell = 10$ and therefore $\ell = 8$. So the domain of $N(\ell)$ is $0 \le \ell \le 8$.

$$N(\ell) = \underline{\quad 10 - 1.25 \cdot \ell \quad}, \text{ with domain } \underline{\quad 0 \le \ell \le 8 \quad}$$