7. [10 points] Emanuel grows corn on his farm to produce corn oil, and is hoping to increase his income by producing more corn oil. Unfortunately, Emanuel has to be careful about exactly how many corn plants he grows: if he plants too much corn, he risks crowding the plants and causing some of them to wilt and die. However, he knows that if he plants an additional p tons of corn, he can produce a total of $C(p)$ gallons of corn oil, where $C(p)$ is given by the formula:

$$C(p) = -3p^2 + \sqrt{11}p + 200.$$

a. [3 points] Find the average rate of change of $C(p)$ on the interval $1 \leq p \leq 5$. Your answer can be exact or accurate to three decimal places, but should **include units**.

Solution: The average rate of change is

$$\frac{C(5) - C(1)}{5 - 1} = \frac{-72 + 4\sqrt{11}}{4} \approx -14.683$$

The average rate of change is -14.683 gallons/ton

b. [5 points] Write $C(p)$ in vertex form by completing the square. Your answer must be **exact**, and you must **show all your work**, **step-by-step**, to get full credit.

Solution: By completing the square, we get:

$$\begin{aligned} C(p) &= -3p^2 + \sqrt{11}p + 200 \\ &= -3\left(p^2 - \frac{\sqrt{11}}{3}p\right) + 200 \\ &= -3\left(p^2 - \frac{\sqrt{11}}{3}p + \frac{11}{36} - \frac{11}{36}\right) + 200 \\ &= -3\left(p^2 - \frac{\sqrt{11}}{3}p + \frac{11}{36}\right) + \frac{11}{12} + 200 \\ &= -3\left(p - \frac{\sqrt{11}}{6}\right)^2 + \left(200 + \frac{11}{12}\right) \end{aligned}$$

$C(p) = -3\left(p - \frac{\sqrt{11}}{6}\right)^2 + \left(200 + \frac{11}{12}\right)$

c. [2 points] Based on your answer above, how much corn should Emanuel add to maximize his corn oil production? Your answer should be **exact** and **include units**.

Emanuel should add $\frac{\sqrt{11}}{6}$ tons of corn