

2. [11 points] Gretchen’s experiments have quickly depleted Chuck’s egg supply, and he has to buy more eggs from the wholesaler. Chuck has \$1050 to spend, and the wholesaler informs him that the price of mealworm eggs and cricket eggs are now \$6.50 and \$8 per pound, respectively.
- a. [4 points] Let f be the function that gives the amount M of mealworm eggs in pounds that Chuck can afford if he buys C pounds of cricket eggs (in other words, we have $M = f(C)$). Write a formula for the function f .

Solution: We have $6.5M + 8C = 1050$. Rearranging gives $M = \frac{2}{13}(1050 - 8C)$.

$$f(C) = \frac{2}{13}(1050 - 8C)$$

After some bargaining, the wholesaler gives Chuck a **special offer**. If he buys \$400 worth of cricket eggs at \$8 per pound, then he will be charged only \$7.50 for each subsequent pound of cricket eggs beyond the first \$400.

- b. [2 points] If Chuck spends \$400 on cricket eggs, what amount of mealworm eggs can he buy? Circle your final answer.

Solution: If Chuck spends \$400 on cricket eggs, he buys 50 pounds of them. We compute $f(50) = 2/13 \cdot (1050 - 400) = 100$. He can buy 100 pounds of mealworm eggs.

- c. [5 points] Let g be the new function that gives the amount M of mealworm eggs in pounds that Chuck can afford if he buys C pounds of cricket eggs with the **special offer**. Write a piecewise-defined formula for the function g .

Solution: There is no change in price if Chuck buys less than 50 pounds of cricket eggs, so $g(C) = f(C)$ for $0 \leq C \leq 50$. Thereafter, the function is linear with slope $-7.5/6.5 = -15/13$. By the previous part, we know that the graph passes through the point $(50, 100)$. The formula on this stretch is $g(C) = -15/13 \cdot (C - 50) + 100$. Finally, we need to find the right endpoint for the domain of g :

$$\frac{-15}{13}(C - 50) + 100 = 0$$

$$C = \frac{1300}{15} + 50 = \frac{410}{3}$$

$$g(C) = \begin{cases} f(C) & \text{for } 0 \leq C \leq 50 \\ -15/13 \cdot (C - 50) + 100 & \text{for } 50 \leq C \leq 410/3 \end{cases}$$