7. [15 points] In one of his experiments, David recorded the speeds (in km/sec) of two different particles, particle A and particle B, for 8 seconds.
Let $S(t)$ be the difference between the recorded speeds of the two particles (in km/sec) t seconds after the beginning of the experiment, i.e. $S(t)$=(speed of particle A)-(speed of particle B).
David found that $S(t) = -\frac{5}{8}t^2 + 5t - 4$.

a. [5 points] Find **both coordinates** of the *maximum* of $S(t)$ by completing the square. Show your work step-by-step.

Solution:

$$\begin{aligned} S(t) &= -\frac{5}{8}t^2 + 5t - 4 \\ &= -\frac{5}{8}(t^2 - 8t) - 4 \\ &= -\frac{5}{8}(t^2 - 8t + 16 - 16) - 4 \\ &= -\frac{5}{8}(t^2 - 8t + 16) + 10 - 4 \\ &= -\frac{5}{8}(t-4)^2 + 6 \end{aligned}$$

$S(t)$ has a *maximum* at (4,6).

b. [4 points] Find all t-values when the speeds of the two particles are equal to each other. Be sure to show your work and give you answer in **exact** form.

Solution: $S(t) = 0 \Rightarrow -\frac{5}{8}t^2 + 5t - 4 = 0$. Using the quadratic formula, we get $t_{1,2} = \frac{-5 \pm \sqrt{25 - 4(-4)\left(-\frac{5}{8}\right)}}{2\left(-\frac{5}{8}\right)} = \frac{-5 \pm \sqrt{15}}{-\frac{5}{4}} = 4 \pm \frac{4}{5}\sqrt{15}$.

c. [3 points] The average rate of change of $S(t)$ between $t = 2$ and $t = 5$ is $0.625\ \frac{\text{km/sec}}{\text{sec}}$. Give a practical interpretation for this average rate of change.

Solution: Between the second and the fifth second, the difference of the recorded speeds of the two particles increases by 0.625 km/sec every second on average.

d. [3 points] Find all t-values in the practical domain of $S(t)$ when particle B is moving faster than particle A.

Solution: We need to determine the $t-$values for which $S(t) < 0$.
t is in $[0, 4 - \frac{4}{5}\sqrt{15}) \cup (4 + \frac{4}{5}\sqrt{15}, 8]$.