

8. [8 points] Find formulas for the following functions using the information given in each part. Be sure to show how you got your formula in each part. Full credit will only be given to answers with full work shown.

a. [4 points] Suppose  $Q(m)$  has the following properties:

- $Q(m)$  is quadratic.
- The vertex of the graph of  $Q(m - 2) + 5$  is  $(5, 4)$ .
- $Q(-1) = 2$ .

*Solution:* The vertex of the graph of  $Q(m - 2) + 5$  is  $(5, 4)$ . Therefore, the vertex of the graph of  $Q(m)$  is  $(3, -1)$ .

We can write:  $Q(m) = a(m - 3)^2 + 1$ . Since  $Q(-1) = 2$ , we get:

$$2 = a(-1 - 3)^2 + 1, \text{ so } a = \frac{3}{16}.$$

$$Q(m) = \frac{3}{16}(m - 3)^2 + 1$$

b. [4 points] Suppose  $E(m)$  has the following properties:

- $E(m)$  is exponential.
- $\frac{E(-5)}{E(-3)} = \frac{1}{0.7225}$
- $E(-1) = 2$ .

*Solution:* We know that  $E(m) = ab^m$ .

$$\frac{1}{0.7225} = \frac{E(-5)}{E(-3)} = \frac{ab^{-5}}{ab^{-3}}, \text{ so } b^2 = 0.7225.$$

Taking a positive square root (because  $b > 0$ ), we get  $b = \sqrt{0.7225} = 0.85$ .

Since  $E(-1) = 2$  we get:

$$a(0.85)^{-1} = 2, \text{ so } a = 2 \cdot 0.85 = 1.7$$

$$E(m) = 1.7(0.85)^m$$