9. [15 points] Below is a graph of a function $J(w)$ and a table of values for a function $T(z)$. The grid on the graph is made up of squares of side length one.

\[ \begin{array}{c|cccccc}
  z & -3 & -2 & 3 & 4 & 9 \\
  T(z) & 9 & 3 & 1 & 3 & c \\
\end{array} \]

a. [3 points]
Suppose the average rate of change of $T(z)$ between $z = -3$ and $z = 9$ is 2.5. Find $c$.

**Solution:**

\[
\frac{c - 9}{9 - (-3)} = 2.5
\]

\[
c = 39
\]

b. [4 points]
Find all solutions to the equation $T(J(w)) = 3$ using only the information about $J(w)$ and $T(z)$ above. Find exact answers if possible, or estimate using the grid if needed. Circle your final answer(s).

**Solution:** We look at which inputs of $T(z)$ are needed to get output 3. From the table, we need an input of -2 or 4, hence we need to look at what inputs of $J(w)$ give those outputs. From the graph, we see that we get two valid inputs

\[ w = -4, 0 \]

c. [8 points]

$J(w)$ is comprised of a linear piece and a quadratic piece. Find a piecewise-defined function for $J(w)$. Circle your answer.

**Solution:** The piecewise function consists of a linear part followed by a quadratic part. From the graph, the linear part goes between $(-4, -2)$ and $(0, 4)$, so we deduce that the linear function is $\frac{3}{2}w + 4$. For the quadratic part, we have two zeros at $w = 1$ and $w = 3$, so we put it into factored form $a(w - 1)(w - 3)$. Using the point $(0, 4)$, we deduce that $a = \frac{4}{3}$. Putting them together into the a piecewise function gives

\[
J(w) = \begin{cases} 
\frac{3}{2}w + 4 & -4 \leq w < 0 \\
\frac{4}{3}(w - 1)(w - 3) & 0 \leq w < 4 
\end{cases}
\]