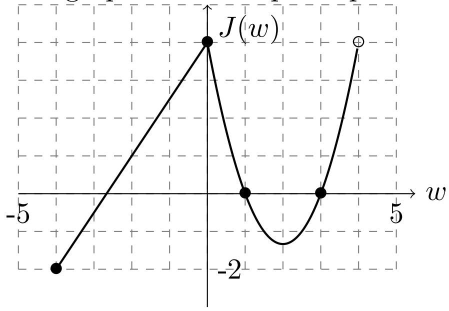
9. [15 points] Below is a graph of a function $J(w)$ and a table of values for a function $T(z)$. The grid on the graph is made up of squares of side length one.



z	-3	-2	3	4	9
$T(z)$	9	3	1	3	c

a. [3 points]
Suppose the average rate of change of $T(z)$ between $z = -3$ and $z = 9$ is 2.5. Find c.

Solution:

$$\frac{c-9}{9-(-3)} = 2.5$$
$$c = 39$$

c=______39______.

b. [4 points]
Find all solutions to the equation

$$T(J(w)) = 3$$

using only the information about $J(w)$ and $T(z)$ above. Find exact answers if possible, or estimate using the grid if needed. Circle your final answer(s).

Solution: We look at which inputs of $T(z)$ are needed to get output 3. From the table, we need an input of -2 or 4, hence we need to look at what inputs of $J(w)$ give those outputs. From the graph, we see that we get two valid inputs

$$w = -4,\ 0$$

c. [8 points]
$J(w)$ is comprised of a linear piece and a quadratic piece. Find a piecewise-defined function for $J(w)$. Circle your answer.

Solution: The piecewise function consists of a linear part followed by a quadratic part. From the graph, the linear part goes between $(-4, -2)$ and $(0, 4)$, so we deduce that the linear function is $\frac{3}{2}w + 4$. For the quadratic part, we have two zeros at $w = 1$ and $w = 3$, so we put it into factored form $a(w-1)(w-3)$. Using the point $(0, 4)$, we deduce that $a = \frac{4}{3}$. Putting them together into the a piecewise function gives

$$J(w) = \begin{cases} \frac{3}{2}w + 4 & -4 \le w < 0 \\ \frac{4}{3}(w-1)(w-3) & 0 \le w < 4 \end{cases}$$