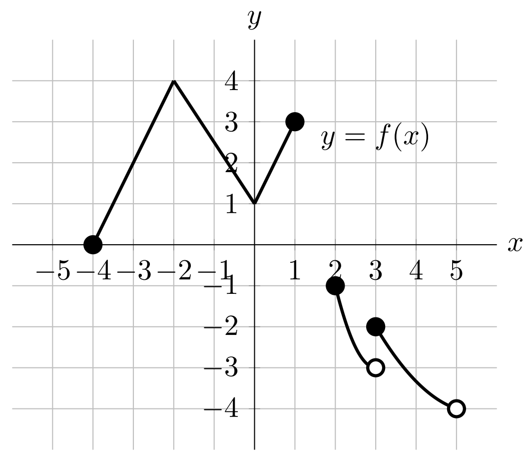
2. [8 points]

The entire graph of a function $y = f(x)$ is given to the right. Note that $f(x)$ is piecewise-linear for $-4 \le x \le 1$.



A different function, $g(x)$, is given in the following table.

x	-3	-2	-1	0	0.2	1	2	3
$g(x)$	5	4	1	0	-1	-2	-4	-3

a. [3 points] Find

(i) the domain of $f(x)$ and

(ii) the range of $f(x)$.

Write your answers using inequalities or interval notation. Make sure to clearly label which answer is the domain and which is the range. You do not need to justify your answer.

Solution: (i) The domain of $f(x)$ is $-4 \le x \le 1$ and $2 \le x < 5$ (or in interval notation: $[-4, 1]$ and $[2, 5)$.)

(ii) The range is $-4 < x \le -2$ and $0 \le x \le 4$ (in interval notation: $(-4, -2]$ and $[0, 4]$.)

b. [5 points] Find the following values. If you make any calculations to find your answers, include those calculations in your submission.

(i) The average rate of change of $f(x)$ from $x = -4$ to $x = 1$

(ii) $g(f(3))$

(iii) All values of x so that $g(f(x)) = -2$

Solution:

(i) The average rate of change of f between $x = -4$ and $x = 1$ is

$$\frac{f(1) - f(-4)}{1 - (-4)} = \frac{3 - 0}{5} = \frac{3}{5}.$$

(ii) Since $f(3) = -2$, we have $g(f(3)) = g(-2) = 4$.

(iii) If the output of g is -2, the input of g must be 1, and so if $g(f(x)) = -2$, then $f(x) = 1$. So, we just need to solve $f(x) = 1$ for x. From the graph, we see there are two values where this happens:

$$x = -3.5 \quad \text{and} \quad x = 0.$$

(To see why $x = -3.5$ is a solution: we know $f(x)$ is linear between $x = -4$ and $x = -2$, and has slope 2. Since $f(-4) = 0$, if we want the output to go up 1, the input needs to increase by .5, yielding $x = -3.5$.)