**5.** [13 points] You're buying baking supplies (flour, butter, apples, etc.) for your pie business. Let $A(d)$ be the amount of money (in dollars) you'll get back from selling the pies you make from $d$ dollars of supplies. For each dollar you spend on supplies between 0 and 50 dollars, you'll get back \$2. For each additional dollar over the first 50 dollars you invest you get back \$3.

**a**. [4 points] From the description above, write down a piecewise formula for the function $A(d)$. Use standard piecewise notation:

$$A(d) = \begin{cases} \\ \\ \end{cases}$$

*Solution:* For the first 50 dollars, you get back 2 dollars for each dollar you invest, so the formula is just $A(d) = 2d$ on that interval. After the first 50, you get 3 dollars for every dollar you invest. So, if you spend $d \geq 50$ dollars, you get:

- 2 dollars for each of the first 50 dollars you spend, or 100 dollars, and
- 3 dollars for each dollar beyond the first 50 dollars, that is, $3(d-50)$ dollars.

So, the piecewise formula is given by

$$A(d) = \begin{cases} 2d & 0 \leq d \leq 50 \\ 3(d-50)+100 & 50 \leq d \end{cases}$$

**b**. [4 points] Evaluate $A^{-1}(190)$ and give a practical interpretation of your answer.

*Solution:* To make 190 dollars, it's clear you have to invest more than 50 dollars (as this would only get you 100 dollars). So, you need to invest the first 50 (receiving 100 back), and then you need to make 90 more; since each dollar after the first 50 dollars returns 3, you just need to invest $90/3 = 30$ more dollars, for a total of 80 dollars.
A practical interpretation of the answer is "To get 190 dollars back in sales, you need to invest 80 dollars in supplies".

**c**. [5 points] Find a piecewise formula for the composition $A(A(d))$. Use standard piecewise notation.

*Solution:* For $d \leq 50$, we have $A(d) = 2d$, and when $2d \leq 50$ we then have $A(A(d)) = A(2d) = 4d$. This stops being valid when $2d \geq 50$, that is, when $d \geq 25$.
When $25 \leq d \leq 50$, you'll still have $A(d) = 2d$, but since $2d \geq 50$ you have $A(A(d)) = A(2d) = 3(2d-50)+100$.
Finally, when $d \geq 50$, we have $A(d) = 3(d-50)+100 = 3d-50$; since this is $\geq 50$, we have

$$A(A(d)) = A(3d-50) = 3((3d-50)-50)+100 = 3(3d-100)+100.$$

Thus, we have that

$$A(A(d)) = \begin{cases} 4d & 0 \leq d \leq 25 \\ 3(2d-50)+100 & 25 \leq d \leq 50 \\ 3(3d-100)+100 & 50 \leq d. \end{cases}$$