5. [9 points] Danica is tossing her wadded-up notes into a wastebasket across the room. The height above the ground of one particular wad, H (in feet), can be expressed as a function of the horizontal distance, d (in feet), from where Danica releases the wad of paper using the following function:

$$H = -\frac{1}{4}\left(d^2 - 4d - 12\right)$$

a. [2 points] Find the value of the vertical intercept and interpret its meaning in the context of the problem.

Answer: Vertical intercept: _____3_____

Interpretation:

Solution: When Danica releases the wad of paper, it is 3 feet from the ground.

b. [3 points] The rim of the wastebasket is 1.5 feet above the ground and 7 feet horizontally from where Danica released the wad of paper. Using this information, can you tell whether Danica succeeds in throwing the wad into the wastebasket?
Show all calculations and justify your conclusion with one sentence.

Answer *(circle one)*:   She succeeds | She fails | Cannot be determined

Justification:

Solution: There are a few ways to reach this conclusion. One is to use the formula to find that when $d = 7$, $H = -2.25$. The fact that the height is negative tells us that the wad of paper has already hit the floor earlier, and this is no longer a good model for the height of the paper. Another way is to factor the formula to find $H = -\frac{1}{4}(d+2)(d-6)$. This tells us that the paper hits the floor 6 feet away from Danica, a full foot short of the wastebasket.

c. [4 points] What is the highest point above the ground the wad reaches? Include units.
There are at least two methods you could use here: finding the axis of symmetry using the zeros or completing the square.

Solution: This question is asking us to find the value of H at the vertex of the quadratic function. In the previous part, we found that the zeroes are at $d = -2$ and $d = 6$. This means that the vertex is exactly halfway in between, at $d = 2$. Now we can plug this into the formula to find that $H = \frac{1}{4}(2^2 - 4 \cdot 2 - 12) = 4$ feet when $d = 2$.

Answer: _____4 feet_____