1. [10 points] You are looking to model the growth of a new TikTok hashtag #Math105FUN and you have some data to help you. Initially, at time \( t = 0 \), there are 100 videos with this hashtag. Ten days later (at time \( t = 10 \)), there are 500 videos with this hashtag.

a. [3 points] If you assume the growth of this hashtag is linear, find an expression for the function \( L(t) \) giving the number of videos with the hashtag #Math105FUN as a function of \( t \) given in days. Your function should match the data points you have so far.

\[
L(t) = 100 + 40t
\]

b. [3 points] If you assume, instead, that the growth of this hashtag is exponential, find an expression for the function \( E(t) \) giving the number of videos with the hashtag #Math105FUN as a function of \( t \) given in days. Your function should match the data points you have so far.

\[
E(t) = 100 \cdot (5^{\frac{1}{10}})^t
\]

Solution: We are told the function is exponential, and we know its initial value is 100. We need to find its growth factor. Since it grows by a factor of 5 in 10 hours, it will grow by a factor of \( 5^{\frac{1}{10}} \) each hour. Putting that together we get the formula below.

\[
E(t) = 100 \cdot (5^{\frac{1}{10}})^t
\]

c. [2 points] You later get another piece of data: at day \( t = 12 \), the number of videos with the hashtag is 690. Which model—\( L(t) \) vs. \( E(t) \)—better fits this new information? Show all work.

Solution: We can plug \( t = 12 \) into both our models to see which output is closer to 690.

\[
L(12) = 100 + 40(12) = 580
\]

\[
E(12) = 100 \cdot (5^{\frac{1}{10}})^{12} = 689.86
\]

From this we see that this new data means that \( E(t) \) is a better fit.

(Circle one) \( L(t) \) is a better fit \( E(t) \) is a better fit

d. [2 points] Let \( H(t) \) denote the total number of videos with a different hashtag — #Math105studyfest — \( t \) days after September 20, 2023. We want a new function \( G(s) \) that instead denotes the total number of #Math105studyfest videos \( s \) days after September 30, 2023. How can we write \( G(s) \) in terms of \( H(t) \)?

\[ G(s) = \ldots \] (Circle the best answer)
Solution:
One way to see this is to notice that when we compute $G(0)$ we should get the number of videos with the hashtag on September 30, so $H(10)$. This concrete point helps us to see that we want $G(s) = H(s + 10)$.

\[
\begin{array}{cccc}
H(s - 10) & H(s + 10) & H(s) + 10 & H(s) - 10 \\
\end{array}
\]