1. [10 points] You are looking to model the growth of a new TikTok hashtag `#Math105FUN` and you have some data to help you. Initially, at time $t = 0$, there are 100 videos with this hashtag. Ten days later (at time $t = 10$), there are 500 videos with this hashtag.

a. [3 points] If you assume the growth of this hashtag is linear, find an expression for the function $L(t)$ giving the number of videos with the hashtag `#Math105FUN` as a function of t given in days. Your function should match the data points you have so far.

$L(t) =$ $100 + 40t$

b. [3 points] If you assume, instead, that the growth of this hashtag is exponential, find an expression for the function $E(t)$ giving the number of videos with the hashtag `#Math105FUN` as a function of t given in days. Your function should match the data points you have so far.

Solution: We are told the function is exponential, and we know its initial value is 100. We need to find its growth factor. Since it grows by a factor of 5 in 10 hours, it will grow by a factor of $5^{\frac{1}{10}}$ each hour. Putting that together we get the formula below.

$E(t) =$ $100 \cdot (5^{\frac{1}{10}})^t$

c. [2 points] You later get another piece of data: at day $t = 12$, the number of videos with the hashtag is 690. Which model—$L(t)$ vs. $E(t)$—better fits this new information? *Show all work.*

Solution: We can plug $t = 12$ into both our models to see which output is closer to 690.

$$L(12) = 100 + 40(12) = 580$$

$$E(12) = 100 \cdot (5^{\frac{1}{10}})^{12} = 689.86$$

From this we see that this new data means that $E(t)$ is a better fit.

(Circle one) $L(t)$ IS A BETTER FIT $E(t)$ IS A BETTER FIT

d. [2 points] Let $H(t)$ denote the total number of videos with a different hashtag — `#Math105studyfest` — t days after September 20, 2023. We want a new function $G(s)$ that instead denotes the total number of `#Math105studyfest` videos s days after September 30, 2023. How can we write $G(s)$ in terms of $H(t)$?

$G(s) = \ldots$ (Circle the best answer)

Solution:
One way to see this is to notice that when we compute $G(0)$ we should get the number of videos with the hashtag on September 30, so $H(10)$. This concrete point helps us to see that we want $G(s) = H(s+10)$.

$H(s-10)$ $\boxed{H(s+10)}$ $H(s)+10$ $H(s)-10$