3. [12 points] A heater is turned on in a cold room. Let $n = f(T)$ be the number of hours it takes for the heater to warm the room to a temperature of T degrees Fahrenheit (°F). A table of values of this function is given below.

T	61	64	66	67	68
$n = f(T)$	0.5	1.3	2.3	3.3	7

The cost, C, in dollars, to run the heater for n hours is given by the formula

$$C = g(n) = 0.25 + 0.4n.$$

Both f and g are invertible functions.

a. [2 points] Compute the quantities $f^{-1}(0.5)$ and $g(f(68))$.

Solution: $g(f(68)) = g(7) = 0.25 + 0.4(7) = 0.25 + 2.8 = 3.05$

Answer: $f^{-1}(0.5) =$ **61** and $g(f(68)) =$ **3.05**

b. [2 points] Find a formula for g^{-1} in terms of C.

Solution: To get the inverse we can solve $C = 0.25 + 0.4n$ for n as follows:

$$C - 0.25 = 0.4n$$

$$\frac{C - 0.25}{0.4} = n$$

This gives us a formula for n in terms of C, which is our inverse.

Answer: $g^{-1}(C) = \frac{C-0.25}{0.4}$

c. [3 points] For each part below, write a phrase or sentence giving a practical interpretation of the given expression or equation, or explain why it doesn't make sense in this context.

i. $g(1) = 0.65$

Solution: The cost to run the heater for 1 hour is \$0.65.

ii. $f(g(3))$

Solution: This composition does not having a meaning in this context. The units of $g(3)$ are dollars, which does not make sense to plug into f, which takes a temperature in degrees Fahrenheit.

(Problem continues on the next page.)

(The problem has been restated here for convenience.)

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Both f and g are invertible functions.

d. [3 points] For each item below, write an expression or equation, possibly involving the functions f, g, and/or their inverses, that represents the given statement.

i. It takes an hour to heat the room to 63 °F.

Solution: $f(63) = 1$ OR $f^{-1}(1) = 63$.

ii. the temperature of the room when the heating costs have reached \$1

Solution: $f^{-1}(g^{-1}(1))$. It is also fine if you solve for $g^{-1}(1)$ explicitly and get instead: $f^{-1}(0.75/0.4)$ or $f^{-1}(1.875)$

e. [2 points] Circle the numeral of the one description below that is best supported by the evidence in this problem. Clearly show your work in the space below.

i. Each °F increase in temperature takes the same amount of time.

$\boxed{\text{ii.}}$ As the room warms up, it takes an increasing amount of time to heat the room to each additional °F in temperature.

iii. It takes less and less time for the heater to heat the room to each additional °F in temperature.

Work:

Solution: Let's use the values in the table to look at how the average rates of change change.

$$\frac{1.3 - 0.5}{64 - 61} = \frac{.8}{3} = 0.2\overline{6} \qquad \frac{2.3 - 1.3}{66 - 64} = \frac{1}{2} = 0.5 \qquad \frac{3.3 - 2.3}{67 - 66} = 1 \qquad \frac{7 - 3.3}{68 - 67} = 3.7$$

Since these average rates of change are increasing, that indicates that the number of additional hours for each additional degree Fahrenheit of heating is getting larger.