3. [12 points] A heater is turned on in a cold room. Let \( n = f(T) \) be the number of hours it takes for the heater to warm the room to a temperature of \( T \) degrees Fahrenheit (°F). A table of values of this function is given below.

<table>
<thead>
<tr>
<th>( T )</th>
<th>61</th>
<th>64</th>
<th>66</th>
<th>67</th>
<th>68</th>
</tr>
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<tbody>
<tr>
<td>( n = f(T) )</td>
<td>0.5</td>
<td>1.3</td>
<td>2.3</td>
<td>3.3</td>
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The cost, \( C \), in dollars, to run the heater for \( n \) hours is given by the formula

\[
C = g(n) = 0.25 + 0.4n.
\]

Both \( f \) and \( g \) are invertible functions.

a. [2 points] Compute the quantities \( f^{-1}(0.5) \) and \( g(f(68)) \).

\[
\text{Solution: } g(f(68)) = g(7) = 0.25 + 0.4(7) = 0.25 + 2.8 = 3.05
\]

Answer: \( f^{-1}(0.5) = 61 \) and \( g(f(68)) = 3.05 \)

b. [2 points] Find a formula for \( g^{-1} \) in terms of \( C \).

\[
\text{Solution: } \text{To get the inverse we can solve } C = 0.25 + 0.4n \text{ for } n \text{ as follows:}
\]

\[
C - 0.25 = 0.4n
\]

\[
\frac{C - 0.25}{0.4} = n
\]

This gives us a formula for \( n \) in terms of \( C \), which is our inverse.

Answer: \( g^{-1}(C) = \frac{C - 0.25}{0.4} \)

c. [3 points] For each part below, write a phrase or sentence giving a practical interpretation of the given expression or equation, or explain why it doesn’t make sense in this context.

i. \( g(1) = 0.65 \)

\[
\text{Solution: } \text{The cost to run the heater for 1 hour is $0.65.}
\]

ii. \( f(g(3)) \)

\[
\text{Solution: } \text{This composition does not having a meaning in this context. The units of } g(3) \text{ are dollars, which does not make sense to plug into } f, \text{ which takes a temperature in degrees Fahrenheit.}
\]

(Problem continues on the next page.)
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\[
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\]

Both \( f \) and \( g \) are invertible functions.

d. [3 points] For each item below, write an expression or equation, possibly involving the functions \( f \), \( g \), and/or their inverses, that represents the given statement.

i. It takes an hour to heat the room to 63 °F.

Solution: \( f(63) = 1 \) OR \( f^{-1}(1) = 63 \).

ii. The temperature of the room when the heating costs have reached $1

\[
\text{Solution: } f^{-1}(g^{-1}(1)). \text{ It is also fine if you solve for } g^{-1}(1) \text{ explicitly and get instead: } f^{-1}(0.75/0.4) \text{ or } f^{-1}(1.875)
\]

e. [2 points] Circle the numeral of the one description below that is best supported by the evidence in this problem. Clearly show your work in the space below.

i. Each °F increase in temperature takes the same amount of time.

[ ]

ii. As the room warms up, it takes an increasing amount of time to heat the room to each additional °F in temperature.

[ ]

iii. It takes less and less time for the heater to heat the room to each additional °F in temperature.

[ ]

Work:
### Solution:

Let’s use the values in the table to look at how the average rates of change change.

\[
\begin{align*}
1.3 - 0.5 &= \frac{0.8}{3} = 0.26 \\
2.3 - 1.3 &= \frac{1}{2} = 0.5 \\
3.3 - 2.3 &= \frac{1}{2} = 0.5 \\
7 - 3.3 &= \frac{3.7}{7 - 3.3} = 3.7
\end{align*}
\]

Since these average rates of change are increasing, that indicates that the number of additional hours for each additional degree Fahrenheit of heating is getting larger.