5. [11 points] A farmer is planning to plant a small apple orchard. She knows that the more trees she plants, the fewer apples each tree will produce due to the effects of crowding. In particular, if she plants x trees, when they are fully grown she expects each tree to produce a kilograms (kg) of apples each year, where

$$a = 20 - \frac{1}{5}x.$$

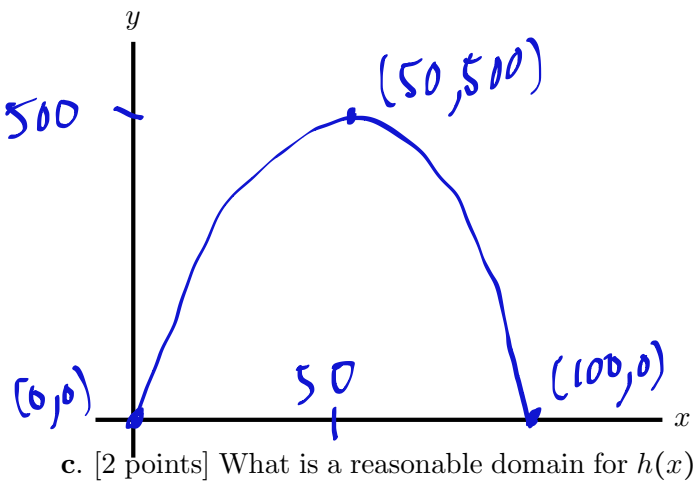
a. [2 points] Describe the meaning of the slope of this line in the context of this problem.

Solution: For each additional tree planted, the expected production from each tree will decrease by $\frac{1}{5}$ kilograms.

Once the trees are fully grown, the total yearly harvest of the farmer's orchard, in kilograms, will be given by the quadratic function

$$h(x) = x\left(20 - \frac{1}{5}x\right).$$

b. [5 points] Sketch a graph of $y = h(x)$ on the axes below. Be sure that the scale of each axis is clear, and that the vertex, vertical intercept, and any zeroes are clear. Also write the (x, y) coordinates of these points in the included blanks.



vertical intercept = $(0,0)$

zeroes = $(0,0)$ **and** $(100,0)$

vertex = $(50,500)$

c. [2 points] What is a reasonable domain for $h(x)$ given the context of the problem? Briefly explain.

Solution: A reasonable domain for is function is $[0, 100]$. It doesn't make sense to produce *less* than 0 trees. And once we're at more than 100 trees, the total apple production is negative, which doesn't make sense.

d. [2 points] How many trees should the farmer plant to maximize her harvest? Briefly explain.

Solution: The farmer should plan 50 trees to maximize her harvest. We can see in our graph of $h(x)$ that the highest total production is achieved at the vertex of the parabola, which is at $(50, 500)$.