3. [12 points]

- The amount of money Bizzi earns in one week, W (in US Dollars), is a function of the number of hours she works that week, h. That is, $W = f(h)$.
- On the other hand, the number of hours she can spend watching K-pop videos, K, is *also* a function of the number of hours she works that week, h. That is, $K = g(h)$.

Both $W = f(h)$ and $K = g(h)$ are invertible functions.

a. [6 points] Describe the meaning of each of the following expressions or equations in the context of Bizzi's life, or explain why the expression or equation doesn't make sense in context.

(i) $f^{-1}(1210) = 50$

Solution: Bizzi must work 50 hours in a week to earn \$1210 that week.
To see this, note that the function f takes as input the number of hours Bizzi works in a week and outputs the amount of money she earns that week, in dollars. Therefore, the inverse function f^{-1} should take as input the amount Bizzi earned, in dollars, in one week and output the number of hours she worked that week.

(ii) $g(f(30)) = 2$

Solution: This composition does not make sense in the context of the problem. The output of f is an amount of money that Bizzi earns in a week. Therefore, $f(30)$ represents some amount of money, in dollars. However, g takes an input a number of hours, not an amount of money in dollars. Therefore, it does not make sense to plug $f(30)$ into g in this context.

(iii) $g(40)$

Solution: This represents the number of hours Bizzi can spend watching K-pop videos if she works 40 hours in a week.

b. [6 points] For each of the following phrases or sentences, write an expression or equation to represent it symbolically using the functions f, g, or their inverses.

(i) Bizzi does *not* work on January 1, but gets paid that week as if she worked 8 hours that day. Express the amount of money Bizzi earns the week of January 1 if she works h hours total during the remainder of the week.

Solution: $f(h+8)$. Bizzi gets paid for the h hours she actually worked plus an additional 8 hours.

(ii) The week of April 1, every employee in the company gets a \$200 bonus. Express the amount of money Bizzi earns the week of April 1 if she works h hours.

Solution: $f(h) + 200$. Bizzi gets paid an additional \$200 on top of the $f(h)$ dollars she would normally get paid for working h hours.

(iii) Represent the following sentence as an equation: On a week where Bizzi earns 950 dollars, she has time to watch 3 hours of K-Pop videos.

Solution: $g(f^{-1}(950)) = 3$ OR $f(g^{-1}(3)) = 950$.
Looking at units, we see that 950 must be an output of f or input of f^{-1} and 3 must be an output of g or input of g^{-1}. To actually translate this into an equation, we note that if Bizzi earns 950 dollars in a week, she must have worked $f^{-1}(950)$ hours. This would give her $g(f^{-1}(950))$ hours to watch K-pop videos, so $g(f^{-1}(950)) = 3$. Alternatively, if Bizzi had 3 hours to watch K-pop videos in a week, she must have worked $g^{-1}(3)$ hours. This would mean she earned $f(g^{-1}(3))$ dollars, so $f(g^{-1}(3)) = 950$.