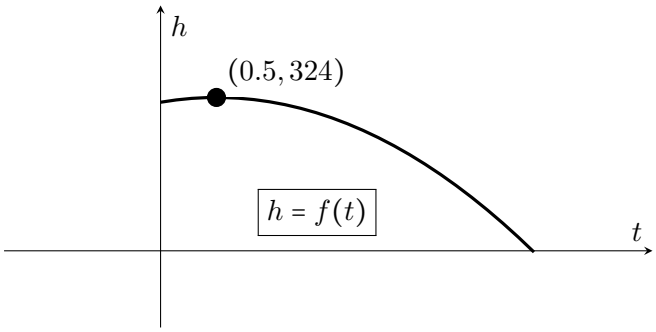
4. [10 points] The AT&T Building in Detroit (photo below, left) is 320 feet tall. A ball is (safely) launched from the level of the roof of the AT&T Building. Its height above the ground h, in feet, is a quadratic function of time t, in seconds. A graph of $h = f(t)$ is shown below, to the right. The parabola's vertex is the point $(0.5, 324)$.



a. [2 points] Given the context of this problem, what is the vertical intercept of the graph $h = f(t)$? Explain how you know.

Vertical intercept: **320**

Explanation:

Solution: The AT&T building is 320 feet tall, and the ball is thrown from the top of the building at $t = 0$.

b. [2 points] How many seconds after launch did the ball reach its maximum height and how high was that?

Seconds after launch when maximum height was reached: **0.5 seconds**

Maximum height reached: **324** feet

c. [3 points] Use the information from parts (a) and (b) to find a formula for $h = f(t)$. *Show all work.*

Solution: The function $h = f(t)$ is a quadratic function whose vertex is $(0.5, 324)$. Therefore, when written in vertex form its formula looks like $h = f(t) = a(t-0.5)^2 + 324$ for some constant a. To solve for a, we can use the fact that $f(0) = a(0-0.5)^2 + 324 = 320$. Therefore

$$\begin{aligned} a(-0.5)^2 + 324 &= 320 \\ 0.25a + 324 &= 320 \\ 0.25a &= -4 \\ a &= -16 \end{aligned}$$

so $f(t) = -16(t-0.5)^2 + 324$.

$f(t) =$ $-16(t-0.5)^2 + 324$

This problem continues on the next page.

d. [3 points] A ball is thrown down from a hovercraft cruising above *Jupiter*. The ball's height above Jupiter's surface, h, in feet, is given by:

$$h = -40t^2 - 20t + 560$$

where t is measured in seconds after the ball was released.

From the moment the ball was released, how many seconds did it take for the ball to reach the surface of Jupiter? *Show all work. Give your answer in exact form, or rounded to at least two decimal places.*

Solution: The ball is on the surface of Jupiter when its height h above the surface of Jupiter is 0 feet. Therefore, we are looking for a positive value of t such that $-40t^2 - 20t + 560$ is equal to 0. We can solve for the values of t which make this expression 0 by factoring it:

$$\begin{aligned} -40t^2 - 20t + 560 &= 0 \\ -20(2t + t - 28) &= 0 \\ 2t + t - 28 &= 0 \\ (2t - 7)(t + 4) &= 0 \\ t = \frac{7}{2} \text{ or } t &= -4 \end{aligned}$$

We don't consider $t = -4$ since it would correspond to a time before the ball was thrown, so we see that it took $t = 7/2$ seconds for the ball to reach the surface of Jupiter.
We could also have computed this answer using the quadratic formula with $a = -40$, $b = -20$, and $c = 560$. This would give us

$$\begin{aligned} t &= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(-40)(560)}}{2(-40)} \\ &= \frac{20 \pm \sqrt{400 + 160(560)}}{-80} \\ &= \frac{20 \pm \sqrt{90,000}}{-80} \\ &= \frac{20 \pm 300}{-80} \end{aligned}$$

as the solutions to the equation $-40t^2 - 20t + 560 = 0$. These two solutions simplify to $\frac{20+300}{-80} = \frac{320}{-80} = -4$ and $\frac{20-300}{-80} = \frac{-280}{-80} = \frac{7}{2}$. Taking the positive value of t, $\frac{7}{2}$, gives us the same solution as factoring did above.

7/2=3.5 seconds

5. [7 points] On the axes below, sketch a graph of a **single function** $j(x)$ that satisfies all of the following properties:

- $j(x)$ has zeros at $x = 1$ and $x = 3$.
- The domain of $j(x)$ is $-6 \leq x < \infty$.
- $j(x) \longrightarrow -2$ as $x \longrightarrow \infty$. In other notation: $\lim_{x \to \infty} j(x) = -2$.