6. [9 points] Isabel's friend Kei lives in the next town over. The two friends are curious about how their water bills compare. Let $I(w)$ be the amount, in dollars, Isabel pays for her water bill for a month if she uses w Centum Cubic Feet (CCFs) of water that month. Let $K(w)$ be the amount, in dollars, Kei pays for their water bill for a month if they use w CCFs of water that month. Both functions are linear and their formulas are:

$$I(w) = 4.1w + 25 \qquad K(w) = 4.5w + 15$$

a. [3 points] Find $K^{-1}(33)$ and write a sentence which explains what the value you find means in the context of the problem. *Show all work. Give your answer in exact form or rounded to at least two decimal places.*

Solution: We can find $K^{-1}(33)$ by solving $K(w) = 4.5w + 15 = 33$ for w. Subtracting 15 from both sides gives us $4.5w = 18$. If we divide by 4.5, we find $w = 18/4.5 = 4$.

$K^{-1}(33) =$ **4**

Meaning:

Solution: If Kei's water bill for a month is \$33, they must have used 4 CCFs of water that month.

b. [1 point] If Kei used two more CCFs of water in August than in June, how much more expensive was their August water bill than their June water bill? *You do not need to show any work.*

Kei's August water bill is **9** dollars more than their June water bill.

Solution: Let c be the amount of water, in CCFs, that Kei used in June. This means they used $c+2$ CCFs of water in August. Then their bill in June is $K(c) = 4.5c + 15$, and their bill in August is $K(c+2) = 4.5(c+2) + 15$. The difference is

$$\begin{aligned} K(c+2) - K(c) &= 4.5(c+2) + 15 - (4.5c + 15) \\ &= 4.5c + 4.5(2) + 15 - 4.5c - 15 \\ &= 4.5(2) \\ &= 9 \end{aligned}$$

. Therefore, their August bill is \$9 more than their June water bill.

c. [2 points] What is the amount of water usage (in CCFs) that would cost the same amount under both water bill plans? *Show all work. Give your answer in exact form, or rounded to at least two decimal places.*

Solution: We want to find the amount of water w which makes $I(w) = K(w)$. Therefore, we need to solve $4.1w + 25 = 4.5w + 15$ for w:

$$\begin{aligned} 4.1w + 25 &= 4.5w + 15 \\ 25 &= 0.4w + 15 \\ 10 &= 0.4w \\ w &= \frac{10}{0.4} \\ w &= \frac{100}{4} \\ w &= 25 \end{aligned}$$

25 CCFs

Let $g(t)$ be the number of CCFs of water Kei's household has used t days since the start of June (so $t = 1$ would correspond to 12:00am on June 2nd). Some values of $g(t)$ are displayed in the table below.

t	1	5	7	11
$g(t)$	5	19.5	28	33

d. [3 points] Kei's family went out of town (and therefore didn't use any water at home) for a couple days during June. Based on the table above, during which of the following time periods is most likely that Kei's family went out of town?

Circle the **one** best possible answer. *Show all work and explain why you circled the option you chose.*

June 3rd to June 5th June 6th to June 8th June 9th to June 11th

Explanation:

Solution: We can compute the average rate of change of $g(t)$ on the intervals given in the table to estimate how much water Kei's family used per day, on average, during each of those intervals.

On the first interval from $t = 1$ to $t = 5$, the average rate of change is $\frac{g(5)-g(1)}{5-1} = \frac{19.5-5}{5-1} = \frac{14.5}{4} = 3.625$ CCFs/day. That means that between 12am on June 2nd and 12am on June 6th, Kei's household used an average of 3.625 CCFs of water per day.

On the interval from $t = 5$ to $t = 7$, the average rate of change is $\frac{g(7)-g(5)}{7-5} = \frac{28-19.5}{7-5} = \frac{8.5}{2} = 4.25$ CCFs/day. So, from 12am on June 6th to 12am on June 8th, Kei's household used an average of 4.25 CCFs of water per day.

Finally, on the interval from $t = 7$ to $t = 11$, the average rate of change is $\frac{g(11)-g(7)}{11-7} = \frac{33-28}{11-7} = \frac{5}{4} = 1.25$ CCFs/day. Therefore, from 12am on June 8th to 12am on June 12th, Kei's household used on average 1.25 CCFs of water per day.

We see that Kei's household had the lowest average water usage per day between June 8th and June 12th. Therefore, the most likely option for when their family went out of town is June 9th to June 11th.