4. [11 points] Consider the line ℓ given by the equation $y = -3 + 0.2x$.

a. [3 points] Find the slope and both intercepts of ℓ.

Solution: Since the given equation is already in slope-intercept form, we see that the slope of ℓ is 0.2 and its y–intercept is -3. To find the x–intercept, we solve the equation $0 = -3 + 0.2x$ to find $x = 15$.

slope: 0.2 x**–intercept:** 15 y**–intercept:** -3

b. [3 points] Find an equation for the line that is perpendicular to the line ℓ (above) and passes through the point $(4, -2)$.

Solution: Since the slope of ℓ is 0.2, the slope of this line is $-1/0.2 = -5$. Using the given point $(4, -2)$ and point-slope form, we find $y + 2 = -5(x - 4)$ so $y = -2 - 5(x - 4)$. (Or, simplifying to slope-intercept form, this is $y = 18 - 5x$.)

Answer: $y =$ $-2 - 5(x - 4)$ **or** $y = 18 - 5x$

c. [5 points] Find an equation for the parabola satisfying *both* of the conditions below.

- Its y–intercept is 5.
- Its vertex is the point on the line ℓ (above) where $x = 10$.

Solution: The point on the line ℓ where $x = 10$ has y–coordinate $-3 + 0.2(10) = -1$. Hence $(10, -1)$ is the vertex of this parabola. Using vertex form, an equation is therefore given by $y = a(x - 10)^2 - 1$ for some non-zero constant a.

To find a we use the other piece of information provided.
The y–intercept of the parabola is 5, so the point $(0, 5)$ is on the parabola. Thus we have $5 = a(0 - 10)^2 - 1$ and, solving for a, we find $a = 0.06$. Hence an equation for the parabola is $y = 0.06(x - 10)^2 - 1$. (In standard form, this is $y = 0.06x^2 - 1.2x + 5$.)

Answer: $y =$ $0.06(x - 10)^2 - 1$