

6. [14 points] The number of internet users has increased dramatically since the internet was first introduced. There were 361 million internet users worldwide in December 2000 and 817 million internet users worldwide in December 2004.³

Let $U(t)$ be the number, in millions, of internet users worldwide t years after December 1997.

Remember to show your work carefully. All numbers appearing in your answers should either be in exact form or be accurate to at least three decimal places.

- a. [3 points] Find the average rate of change of $U(t)$ between $t = 3$ and $t = 7$. Include units.

Solution: Note that $t = 7$ in December 2004 and $t = 3$ in December 2000. Hence $U(7) = 817$ and $U(3) = 361$.

The requested average rate of change is therefore $\frac{U(7) - U(3)}{7 - 3} = \frac{817 - 361}{4} = \frac{456}{4} = 114$ million internet users per year.

Answer: 114 million internet users per year

- b. [4 points] Assuming that $U(t)$ is linear, find a formula for $U(t)$.

Solution: If $U(t)$ is linear, then its average rate of change is constant and from part (a) we know that this constant average rate of change of $U(t)$ is 114. Since $U(3) = 361$, we can use point-slope form to find $U(t) - 361 = 114(t - 3)$ or $U(t) = 361 + 114(t - 3)$. (Simplifying to slope-intercept form, this is $U(t) = 19 + 114t$, so $U(0) = 19$. Hence this model predicts that there were 19 million internet users in December 2007.)

Answer: $U(t) =$ $361 + 114(t - 3)$ or $19 + 114t$

According to this model, how many internet users were there in December 1997?

Answer: 19 million

- c. [7 points] Assuming instead that $U(t)$ is exponential, find a formula for $U(t)$.

Solution: If $U(t)$ is exponential, then there are constants a and b so that $U(t) = ab^t$. Using the data provided in the problem statement, we know that $U(3) = 361$ and $U(7) = 817$. Hence $361 = ab^3$ and $817 = ab^7$. Dividing, we see that $\frac{817}{361} = \frac{ab^7}{ab^3}$ so $\frac{817}{361} = b^4$. Therefore $b = \left(\frac{817}{361}\right)^{1/4}$. Using this value of b in one of the equations above, we see that $361 = a \left[\left(\frac{817}{361}\right)^{1/4}\right]^3$ so $a = 361 \left(\frac{361}{817}\right)^{3/4}$. Therefore, a formula for $U(t)$ is $U(t) = 361 \left(\frac{361}{817}\right)^{3/4} \left[\left(\frac{817}{361}\right)^{1/4}\right]^t$. (This model thus predicts that there were $361 \left(\frac{361}{817}\right)^{3/4}$ or about 195.646 million internet users in December 1997.)

Answer: $U(t) =$ $361 \left(\frac{361}{817}\right)^{3/4} \left[\left(\frac{817}{361}\right)^{1/4}\right]^t \approx 195.6460(1.2265^t)$

According to this model, how many internet users were there in December 1997?

Answer: Approximately 195.646 million

³Source: <http://www.internetworldstats.com>