7. [17 points] Passengers on a cruise ship watch as nearby dolphins and porpoises jump through the waves. When one dolphin jumps, its height above the water (measured in feet) t seconds after leaving the water is given by $w(t) = -16t^2 + 96Dt$ for some positive constant D.

a. [3 points] After how many seconds, in terms of D, does this dolphin land back in the water?

Solution: The dolphin lands in the water when its height above the water is 0 ft. So we solve the equation $w(t) = 0$. Factoring, we see that $w(t) = -16t^2+96Dt = -16t(t-6D)$ so $w(t) = 0$ when $t = 0$ and when $t = 6D$. Since $t = 0$ when the dolphin first leaves the water, $t = 6D$ when the dolphin lands back in the water. Hence the dolphin lands back in the water after $6D$ seconds.

Answer: $6D$ **seconds**

b. [7 points] Use the method of completing the square to rewrite $w(t)$ in vertex form. What is the vertex of the graph of $w(t)$? (Carefully show your work step-by step. Your answers may involve D.)

Solution:

$$\begin{aligned} w(t) &= -16t^2 + 96Dt \\ &= -16(t^2 - 6Dt) \\ &= -16(t^2 - 6Dt + (-3D)^2 - (-3D)^2) \\ &= -16\left[(t-3D)^2 - 9D^2\right] \\ &= -16(t-3D)^2 + 144D^2 \end{aligned}$$

Vertex Form: $w(t) =$ $-16(t-3D)^2 + 144D^2$ **Vertex:** $(3D, 144D^2)$

c. [2 points] If the dolphin reaches a maximum height of 16 ft before falling back to the water, find the value of D.

Solution: The maximum height of the dolphin is the second coordinate of the vertex found above. So, $144D^2 = 16$ and thus $D^2 = 16/144 = 1/9$ so $D = \pm 1/3$. Since D is positive, $D = 1/3$.

Answer: $D =$ $1/3$

d. [5 points] A nearby porpoise is also seen jumping. Its height above the surface of the water (measured in meters) t seconds after the *dolphin* left the water is given by $h(t) = -5t^2 + 24t - 26$. For how long is the porpoise above the surface of the water? *Solve for the answer algebraically and give your final answer either in exact form or accurate to at least three decimal places.*

Solution: We solve the equation $h(t) = 0$ to find the time when the porpoise left and the time when the porpoise returned to the water. Using the quadratic formula, we have $-5t^2 + 24t - 26 = 0$ when $t = \frac{-24\pm\sqrt{24^2-4(-5)(-26)}}{2(-5)} = \frac{-24\pm\sqrt{56}}{-10} = \frac{-24\pm 2\sqrt{14}}{-10} = \frac{12}{5} \pm \frac{\sqrt{14}}{-5}$. So, the porpoise leaves the water when $t = \frac{12}{5} - \frac{\sqrt{14}}{-5} \approx 1.6517$ and lands back in the water when $t = \frac{12}{5} - \frac{\sqrt{14}}{-5} \approx 3.1483$. Thus the porpoise is above the water for $(\frac{12}{5} - \frac{\sqrt{14}}{-5}) - (\frac{12}{5} + \frac{\sqrt{14}}{-5}) = \frac{2\sqrt{14}}{5} \approx 1.497$ seconds.

Answer: $\frac{2\sqrt{14}}{5} \approx 1.497$ **seconds**