7. [17 points] Passengers on a cruise ship watch as nearby dolphins and porpoises jump through the waves. When one dolphin jumps, its height above the water (measured in feet) \( t \) seconds after leaving the water is given by \( w(t) = -16t^2 + 96Dt \) for some positive constant \( D \).

a. [3 points] After how many seconds, in terms of \( D \), does this dolphin land back in the water?

\[
\text{Answer: } 6D \text{ seconds}
\]

b. [7 points] Use the method of completing the square to rewrite \( w(t) \) in vertex form. What is the vertex of the graph of \( w(t) \)? (Carefully show your work step-by-step. Your answers may involve \( D \).)

\[
\text{Solution: } w(t) = -16t^2 + 96Dt = -16(t^2 - 6Dt) = -16(t^2 - 6Dt + (-3D)^2 - (-3D)^2) = -16 \left( (t - 3D)^2 - 9D^2 \right) = -16(t - 3D)^2 + 144D^2
\]

\[
\text{Vertex Form: } w(t) = -16(t - 3D)^2 + 144D^2 \quad \text{Vertex: } (3D, 144D^2)
\]

c. [2 points] If the dolphin reaches a maximum height of 16 ft before falling back to the water, find the value of \( D \).

\[
\text{Solution: } \text{The maximum height of the dolphin is the second coordinate of the vertex found above. So, } 144D^2 = 16 \text{ and thus } D^2 = 16/144 = 1/9 \text{ so } D = \pm 1/3. \text{ Since } D \text{ is positive, } D = 1/3.
\]

\[
\text{Answer: } D = \frac{1}{3}
\]

d. [5 points] A nearby porpoise is also seen jumping. Its height above the surface of the water (measured in meters) \( t \) seconds after the dolphin left the water is given by \( h(t) = -5t^2 + 24t - 26 \). For how long is the porpoise above the surface of the water?

\[
\text{Solution: } \text{We solve the equation } h(t) = 0 \text{ to find the time when the porpoise returned to the water. Using the quadratic formula, we have } -5t^2 + 24t - 26 = 0 \text{ when } t = \frac{-24 \pm \sqrt{24^2 - 4(-5)(-26)}}{2(-5)} = \frac{-24 \pm \sqrt{576}}{-10} = \frac{-24 \pm 24\sqrt{14}}{-10} = \frac{12}{5} \pm \sqrt{14/5}.
\]

So, the porpoise leaves the water when \( t = \frac{12}{5} - \sqrt{14/5} \approx 1.6517 \) and lands back in the water when \( t = \frac{12}{5} + \sqrt{14/5} \approx 3.1483 \). Thus the porpoise is above the water for \( (\frac{12}{5} + \sqrt{14/5}) - (\frac{12}{5} - \sqrt{14/5}) = 2\sqrt{14/5} \approx 1.497 \) seconds.

\[
\text{Answer: } 1.497 \text{ seconds}
\]