8. [13 points] The *karat rating* of a gold alloy is defined to be 24 times the concentration of gold in the alloy. That is, the karat rating is $24 \cdot \dfrac{\text{mass of gold in alloy}}{\text{total mass of alloy}}$.

Rose gold is an alloy of gold and copper. A metallurgist is experimenting to see how the color of rose gold changes when more or less copper is added. In each trial, the metallurgist starts with 15 grams of gold and 5 grams of copper and then adds or removes copper to change the composition. Let $K(c)$ be the karat rating of the metallurgist's rose gold if c grams of copper have been added to ($c > 0$) or removed from ($c < 0$) the initial 5 grams.

a. [2 points] Find $K(0)$.

Solution: If no grams have copper have been added, then the rose gold consists of 15 grams of gold and 5 grams of copper, so $K(0) = 24 \cdot \frac{15}{20} = 18$. So the karat rating is 18.

Answer: $K(0) =$ 18

b. [5 points] In the context of this problem, what are the domain and range of $K(c)$? (You may use either interval notation or inequalities to describe the domain and range.) *Show your work and explain your reasoning.*

Solution: Any amount of copper can be added to the sample, but at most 5 grams can be removed. Therefore, in the context of this problem, c can be any number in the interval $[-5, \infty)$. Hence the domain of $K(c)$ is $[-5, \infty)$.
The concentration of gold in the alloy can be anywhere from 0% to 100%, not including 0% since gold is never removed from the sample, but including 100% since the metallurgist could remove all 5 grams of copper. Hence the karat rating of the sample can be anywhere from 0 to 24 (not including 0, but including 24). In other words, the range of $K(c)$ is $(0, 24]$.

Domain: $[-5, \infty)$ **Range:** $(0, 24]$

c. [3 points] Find a formula for $K(c)$.

Solution: The total mass of gold in the sample is fixed at 15 grams, but the total mass of the alloy after c grams of copper have been added or removed is $20 + c$. Thus, a formula for $K(c)$ is $K(c) = 24\dfrac{15}{20 + c} = \dfrac{360}{20 + c}$

Answer: $K(c) =$ $\dfrac{360}{20 + c}$

One variety of *white gold* is an alloy of gold and nickel.
Let $V(k)$ be the value, in dollars per gram, of k karat white gold.

d. [3 points] Give an interpretation, in the context of this problem, of the equation $V^{-1}(14) = 10$. *Use a complete sentence and include units.*

Solution: White gold that has a value of 14 dollars per gram is 10 karat.