5. [10 points] A rocket is launched from the ground. The rocket consumes fuel as it moves away from the ground. As the rocket increases in altitude, the atmospheric pressure decays. Consider the following functions:

i) $H(t)$ is the height of the rocket above the ground, in kilometers, t seconds after being launched.

ii) $P(h)$ is the atmospheric pressure, in inches of mercury, h kilometers above the ground.

iii) $G(t)$ is the total amount of fuel burned by the rocket, in gallons, t seconds after it was launched.

Assume that $H(t)$ and $P(h)$ and $G(t)$ are invertible functions.

a. [8 points] For each of the sentences below, fill in the blank with one expression from the list of possible answers given below that makes the statement true"

Possible answers:

$P^{-1}(H^{-1}(15))$	15	$P(H(15))$	$H^{-1}(P^{-1}(15))$
$G^{-1}(15)$	$H^{-1}(15)$	$H(P(15))$	$H(15)$
$P(15)$	$P^{-1}(15)$	$G(15)$	$\frac{1}{15}$

Solution:

i) After $H^{-1}(15)$ seconds, the rocket's height above ground is **15** kilometers.

ii) The rocket is $\mathbf{H(15)}$ kilometers above the ground after 15 seconds.

iii) The atmospheric pressure around the rocket 15 seconds after the rocket was launched is $\mathbf{P(H(15))}$ inches of mercury.

iv) It takes $\mathbf{G^{-1}(15)}$ seconds for the rocket to burn 15 gallons of fuel.

b. [2 points] Suppose that the constants a and b satisfy $G(H^{-1}(a)) = b$. What are the units of a and b ?

Solution: Units of a: **kilometers** Units of b: **gallons**.