3. [14 points] Consider the functions $H(x)$, $G(x)$ and $M(x)$

x	-1	0	1	2	3
$H(x)$	2	3	-1	0	1

$$G(x) = \begin{cases} x - 2 & -1 < x < 1 \\ 0 & x = 1 \\ x^3 & 1 < x \le 3 \end{cases}$$

$$M(x) = \frac{1 - x}{2x}$$

Assume that the function H has an inverse.

a. [8 points] Find the value of the following mathematical expressions. If the expression is undefined, write UNDEFINED.

> *Solution:*
>
> $$G(1) = 0$$
> $$H^{-1}(2) = -1$$
> $$(M(2))^{-1} = \frac{2(2)}{1 - 2} = -4$$
>
> $$G(H(1)) = G(-1) \quad \text{UNDEFINED}$$
> $$H(3G(0) + 5) = H(3(-2) + 5) = H(-1) = 2$$

b. [3 points] Solve the equation $H(M(x)) = 0$. Show all your algebraic work.

> *Solution:* The output of the function H is equal to zero when the input is equal to two. Then $M(x) = 2$. In this case we have
>
> $$\frac{1 - x}{2x} = 2, \quad 1 - x = 4x \quad \text{then} \quad x = \frac{1}{5}.$$

c. [3 points] What is the average rate of change of $G(x)$ for $-\frac{1}{2} \le x \le 3$. Show all your work.

> *Solution:*
> $$\frac{\Delta G}{\Delta x} = \frac{G(3) - G(-\frac{1}{2})}{3.5} = \frac{27 - (-2.5)}{3.5} = \frac{29.5}{3.5} = 8.428.$$