5. [12 points] A coffee shop owner buys coffee from company A or company B. Let $A(c)$ and $B(c)$ be the cost (in dollars) of buying c pounds of coffee from company A and company B respectively. The formulas for the cost functions are given below

$$A(c) = 15 + 8.25c \qquad \text{and} \qquad B(c) = 22 + 7.85c.$$

a. [3 points] What is the practical interpretation of the slope of $A(c)$?

Solution: The cost in dollars of buying an additional pound of coffee from company A.

In the following questions, you must find all your answers ***algebraically***. Show all your work. Your answers must be accurate up to the first two decimals.

b. [2 points] How many pounds of coffee do you need to buy in order for the cost of the coffee to be the same if you buy it either from company A or company B?

Solution: We need to find the number c of lbs of coffee such that

$$\begin{aligned} 15 + 8.25c &= 22 + 7.85c. \\ 0.40c &= 7 \\ c &= \frac{7}{0.40} = 17.5 \text{ lbs of coffee.} \end{aligned}$$

c. [2 points] If the coffee shop owner wants to buy 1000 dollars worth of coffee from company A, how many pounds of coffee can he afford?

Solution:

$$\begin{aligned} 15 + 8.25c &= 1000. \\ 8.25c &= 985. \\ c &= \frac{985}{8.25} = 119.32 \text{ lbs of coffee.} \end{aligned}$$

d. [5 points] Suppose that the coffee shop owner wants to buy 500 dollars worth of coffee, but he wants to buy 50 percent more coffee from company A than from company B. How many pounds of coffee does he need to buy from company B?

Solution: Let a and b be the number of lbs of coffee that he will buy from company A and B respectively. If he wants to buy 50 percent more coffee from company A than from company B, then $a = 1.5b$. If he spends 500 dlls in coffee, then $A(a) + B(b) = 500$. Therefore

$$\begin{aligned} & A(a) + B(b) = (15 + 8.25a) + (22 + 7.85b) = 500. \\ \text{using } a = 1.5b \quad & 15 + 8.25(1.5b) + (22 + 7.85b) = 500. \\ & 15 + 12.375b + 22 + 7.85b = 500 \\ & 37 + 20.225b = 500 \\ & 20.225b = 463 \qquad b = 22.89 \text{ lbs of coffee from company B.} \end{aligned}$$