6. [17 points] Luis and Elena are two biologists studying the population of frogs and butterflies that live in an island. Upon their arrival to the island, they found that there were 2 thousand frogs in the island. Show all your work.

a. [3 points] Luis believed that the population of frogs living in the island increases by 300 frogs every six months. Let $f(t)$ be the amount of frogs (in thousands) living in the island, t months after they arrived at the island, according to Luis belief. Find a formula for $f(t)$.

Solution: Since $f(t)$ is linear, then $f(t) = mt + b$. We know that the slope m of $f(t)$ is $\frac{0.3}{6}$ (0.3 thousand every month). Since there were 2 thousand frogs in the island when they arrived, then $b = 2$. Hence $f(t) = 2 + 0.05t$.

b. [3 points] Elena's hypothesis is that the population of frogs living in the island increases exponentially at a rate of 23% every month. Let $g(t)$ be the amount of frogs (in thousands) living in the island, t months after they arrived at the island, according to Elena's hypothesis. Find a formula for $g(t)$.

Solution: The function $g(t)$ is exponential, then $g(t) = ab^t$. Since there were 2 thousand frogs in the island when they arrived, then $a = 2$. Since the population of frogs living in the island increases exponentially at a rate of 23% every month, then $b = 1 + 0.23 = 1.23$. Hence $f(t) = 2(1.23)^t$

As the frog's population increased, the amount of butterflies in the island started to decrease. The population of butterflies 2 and 5 months after Elena and Luis arrived at the island was 20 thousand and 7 thousand respectively.

c. [4 points] Let $G(t)$ be a linear function describing the population of butterflies (in thousands) t months after the biologists arrive at the island. Find a formula for $G(t)$.

Solution: The problem states that the points $(2, 20)$ and $(5, 7)$ are in the graph of the linear function $G(t) = mt + b$. The slope of G is $m = \dfrac{7 - 20}{5 - 2} = -\dfrac{13}{3}$.
Using the point $(2, 20)$, we find

$$G(t) = 20 - \frac{13}{3}(t - 2) = -\frac{13}{3}t + \frac{86}{3}.$$

The problem continues on the next page

The statement of the problem has been included for your convenience

As the frog's population increased, the amount of butterflies in the island started to decrease. The population of butterflies 2 and 5 months after Elena and Luis arrived at the island was 20 thousand and 7 thousand respectively.

d. [5 points] Let $H(t)$ be an exponential function describing the population of butterflies (in thousands) t months after the biologists arrive at the island. Find a formula for $H(t)$. Your answer must be in **exact form**.

Solution: The problem states that the points $(2, 20)$ and $(5, 7)$ are in the graph of the exponential function $H(t) = ab^t$. Hence a and b satisfy $20 = ab^2$ and $7 = ab^5$. Then

$$\frac{ab^5}{ab^2} = \frac{7}{20}$$

$$b^3 = \frac{7}{20} = 0.35$$

$$b = \sqrt[3]{0.35}.ab^2 = 20$$

$$a = \frac{20}{(\sqrt[3]{0.35})^2} \qquad f(t) = ab^t = \frac{20}{(\sqrt[3]{0.35})^2}\left(\sqrt[3]{0.35}\right)^t$$

e. [2 points] By what percentage is the population of butterflies reduced every month? Your answer must be accurate up to the first two decimals.

Solution: It is reduced by 29.53% every month.