

7. [11 points] In a small isolated island, the local government has decided to start a recycling program. Consider the following functions:
- Let  $F(r)$  be the amount of money (in millions of dollars) that the local government has to spend in order to recycle  $r$  tons of garbage.
  - Let  $G(p)$  be the amount of recyclable garbage (in tons) the island generates in a year when there are  $p$  thousands of people living in the island.
  - Let  $H(t)$  be the amount of people (in thousands) living in the island  $t$  years after 2010.

Assume that the functions  $F$ ,  $G$  and  $H$  have inverses.

- a. [6 points] Find a practical interpretation to the following mathematical expressions:
- $F(3) = 2$

*Solution:* The government spends 2 million dollars recycling 3 tons of garbage.

- $G(H(4))$

*Solution:* The amount of recyclable garbage (in tons) the island generates in 2014.

- b. [1 point] Let  $A$  be the average rate of change of the function  $G$  for  $3 \leq p \leq 5$ . What are the units of  $A$ ?

*Solution:* Units of  $A$  = tons per thousand of people.

- c. [4 points] Fill in the blanks in the following statements using the correct mathematical expression. A list of possible answers are listed below. Write your own expression if the correct expression is not on the list.

*Solution:*

- The government spends 25 millions of dollars to recycle  $\mathbf{F^{-1}(25)}$  tons of garbage.
- There were  $\mathbf{G^{-1}(F^{-1}(25))}$  thousand people living in the island when the local government spent 25 million dollars recycling garbage.