8. [10 points] A cannon fires a cannonball. Let p be a positive constant and

$$f(t) = -5t^2 + pt + 30$$

be the height of the cannonball (in meters) above the ground t seconds after the cannon was fired.

a. [3 points] Find the value and a practical interpretation of the vertical intercept of the function $f(t)$.

Solution: Vertical intercept: $f(0) = 30$ meters.

Practical interpretation: The height of the cannon above the ground in meters.

b. [5 points] Complete the square to put the formula of f in vertex form. *Carefully show your algebraic work step by step.* Your answer may include the constant p.

Solution:

$$\begin{aligned} f(t) &= -5t^2 + pt + 30 \\ &= -5\left(t^2 - \frac{p}{5}t\right) + 30 \\ &= -5\left[t^2 - \frac{p}{5}t + \left(\frac{p}{10}\right)^2 - \left(\frac{p}{10}\right)^2\right] + 30 \\ &= -5\left[\left(t - \frac{p}{10}\right)^2 - \left(\frac{p}{10}\right)^2\right] + 30 \\ &= -5\left(t - \frac{p}{10}\right)^2 + 5\left(\frac{p}{10}\right)^2 + 30 \\ &= -5\left(t - \frac{p}{10}\right)^2 + \frac{p^2}{20} + 30 = -5\left(t - \frac{p}{10}\right)^2 + \frac{p^2 + 600}{20} \end{aligned}$$

c. [2 points] What should be the value of p if the maximum height of the cannonball is 200 meters above the ground? Find your answer algebraically. Show all your work.

Solution: The maximum height is given by $H_{max} = \dfrac{p^2 + 600}{20}$. Hence the maximum height of the cannonball will be 200 meters if $\dfrac{p^2 + 600}{20} = 200$. Hence $p^2 + 600 = 4000$ and $p = \sqrt{3400}$.