8. [10 points] A cannon fires a cannonball. Let \( p \) be a positive constant and 

\[ f(t) = -5t^2 + pt + 30 \]

be the height of the cannonball (in meters) above the ground \( t \) seconds after the cannon was fired.

a. [3 points] Find the value and a practical interpretation of the vertical intercept of the function \( f(t) \).

\[ \text{Solution: Vertical intercept: } f(0) = 30 \text{ meters.} \]

Practical interpretation: The height of the cannon above the ground in meters.

b. [5 points] Complete the square to put the formula of \( f \) in vertex form. \textit{Carefully show your algebraic work step by step.} Your answer may include the constant \( p \).

\[ f(t) = -5t^2 + pt + 30 = -5 \left( t^2 - \frac{pt}{5} \right) + 30 = -5 \left[ t^2 - \frac{pt}{5} + \left( \frac{p}{10} \right)^2 - \left( \frac{p}{10} \right)^2 \right] + 30 = -5 \left[ (t - \frac{p}{10})^2 - \left( \frac{p}{10} \right)^2 \right] + 30 = -5 \left( t - \frac{p}{10} \right)^2 + 5 \left( \frac{p}{10} \right)^2 + 30 = -5 \left( t - \frac{p}{10} \right)^2 + \frac{p^2}{20} + 30 = -5 \left( t - \frac{p}{10} \right)^2 + \frac{p^2 + 600}{20} \]

c. [2 points] What should be the value of \( p \) if the maximum height of the cannonball is 200 meters above the ground? Find your answer algebraically. Show all your work.

\[ \text{Solution: } \text{The maximum height is given by } H_{\text{max}} = \frac{p^2 + 600}{20}. \text{ Hence the maximum height of the cannonball will be 200 meters if } \frac{p^2 + 600}{20} = 200. \text{ Hence } p^2 + 600 = 4000 \text{ and } p = \sqrt{3400}. \]