9. [12 points] A store sells socks. Let $S(p)$ be the profit (in dollars) the store earns from selling socks at a price of p dollars.

a. [5 points] The store manager notices that if they sell socks at 4 dollars, they get the highest profit of $2,500$ dollars. If they sell socks at 2.50 dollars the profit is 1375 dollars. Suppose $S(p)$ is a quadratic function. Find a formula for $S(p)$.

Solution: The highest point on the graph of the profit function $S(p)$ is at the point $(4, 2500)$. Hence this is the vertex of the quadratic function $S(p)$.Then the vertex form of the function is $S(p) = a(p-4)^2 + 2500$. Since $S(2.5) = 1375$, then $1375 = a(2.5 - 4)^2 + 2500$. This yields $a = -\frac{1125}{(1.5)^2} = -500$. Then $S(p) = -500(p-4)^2 + 2500$.

b. [7 points] The winter season is here and the store is now selling mittens. Let M be the profit (in dollars) the store earns from selling mittens at a price of p dollars, where

$$M = f(p) = -96(p - 3.5)^2 + 600.$$

i) At what price(s) will the store not have any profit from selling mittens? You must find your answer algebraically.

Solution: We need to find the prices p at which $f(p) = -96(p-3.5)^2 + 600 = 0$.

$$\begin{aligned} -96(p-3.5)^2 + 600 &= 0 \\ (p-3.5)^2 &= \frac{600}{96} = 6.25 \\ p - 3.5 &= \pm\sqrt{6.25} = \pm 2.5 \\ p = 6 \qquad & p = 1. \end{aligned}$$

ii) Suppose that the store refuses to sell any mittens at any price at which no profit will be obtained. What is the practical domain and range of the function $f(p)$? Use interval notation or inequalities to answer this question. Your answer needs to be in *exact form* or be accurate up to two decimals.

Solution: The vertex $(3.5, 600)$ represents the maximum point in the graph of $f(p)$. Since the prices will never yield non positive profits, then the practical range of $f(p)$ is $(0, 600]$. The domain is the prices that yield a positive profit. Then the domain of $f(p)$ is $1 < p < 6$.