8. [14 points] The owner of a restaurant has a budget to buy up to 15 hours of advertising time on the radio. She predicts that her profits $P(x)$, in thousands of dollars, when she buys $x$ minutes of advertising on the radio for her restaurant is given by:

$$P(x) = -3x^2 + 40x + 100 \quad \text{ for } \quad 0 \leq x \leq 15.$$

a. [5 points] Write the formula of $P(x)$ in vertex form by completing the square. Show all your work step-by-step to receive full credit.

$$P(x) = \frac{1}{3} \left( -3 \left( x - \frac{20}{3} \right)^2 + \frac{100}{3} \right)$$

b. [3 points] Find the practical domain and range of the function $P(x)$. Your answers must be written in exact form or accurate up to the first two decimals. Use inequalities or interval notation.

Domain: $[0, 15]$  
Range: $[100, 200]$
The statement of the problem has been included below for your convenience.

The owner of a restaurant has a budget to buy up to 15 hours of advertising time on the radio. She predicts that her profits $P(x)$, in thousands of dollars, when she buys $x$ minutes of advertising on the radio for her restaurant is given by:

$$P(x) = -3x^2 + 40x + 100 \quad \text{for} \quad 0 \leq x \leq 15.$$

c. [3 points] What should be the minimum amount of radio advertising time the owner has to buy if she wants to obtain a profit of one hundred fifty thousand dollars?
Your answer should be obtained algebraically and it must be in exact form or accurate up to the first two decimals. Include units. Show all your work.

Answer:______________________________

d. [3 points] Find the average rate of change of the function $P(x)$ for $10 \leq x \leq 15$. Include units. Show all your work.

Answer:______________________________