8. [14 points] The owner of a restaurant has a budget to buy up to 15 hours of advertising time on the radio. She predicts that her profits $P(x)$, in thousands of dollars, when she buys x minutes of advertising on the radio for her restaurant is given by:

$$P(x) = -3x^2 + 40x + 100 \qquad \text{for} \qquad 0 \leq x \leq 15.$$

a. [5 points] Write the formula of $P(x)$ in vertex form by completing the square. Show all your work step-by-step to receive full credit.

$P(x) =$_____

b. [3 points] Find the practical domain and range of the function $P(x)$. Your answers must be written in **exact form** or accurate up to the first two decimals. Use inequalities or interval notation.

Domain:_____ Range: _____

The statement of the problem has been included below for your convenience.

The owner of a restaurant has a budget to buy up to 15 hours of advertising time on the radio. She predicts that her profits $P(x)$, in thousands of dollars, when she buys x minutes of advertising on the radio for her restaurant is given by:

$$P(x) = -3x^2 + 40x + 100 \qquad \text{for} \qquad 0 \leq x \leq 15.$$

c. [3 points] What should be the minimum amount of radio advertising time the owner has to buy if she wants to obtain a profit of one hundred fifty thousand dollars?
Your answer should be obtained **algebraically** and it must be in **exact form** or accurate up to the first two decimals. Include units. Show all your work.

Answer:_____

d. [3 points] Find the average rate of change of the function $P(x)$ for $10 \leq x \leq 15$. Include units. Show all your work.

Answer:_____