5. [14 points] A small country decides to reduce the amount of electrical power they produce using coal. The electrical power W generated with coal in 2008 and 2011 was 250 and 120 megawatts, respectively.

a. [8 points]

i) Suppose that $W = f(t)$, where the function f is exponential and t represents the number of years after 2004. Find a formula for $f(t)$. Your answer must be in **exact form**. Show all your work.

Solution: If $f(t) = ab^t$, then

$$ab^4 = 250 \qquad \text{and } ab^7 = 120$$

$$\frac{ab^7}{ab^4} = b^3 = \frac{12}{25} \qquad \text{then} \qquad b = \left(\frac{12}{25}\right)^{\frac{1}{3}}. \quad \text{Hence } a = \frac{250}{b^4} = \frac{250}{\left(\frac{12}{25}\right)^{\frac{4}{3}}}.$$

Therefore $f(t) = \dfrac{250}{\left(\frac{12}{25}\right)^{\frac{4}{3}}}\left(\dfrac{12}{25}\right)^{\frac{t}{3}} = 250\left(\dfrac{12}{25}\right)^{\frac{t-4}{3}}$

ii) Find the value of the vertical intercept of the function $W = f(t)$ and give a practical interpretation of your answer. Your answer must be **exact** or include at least 2 decimals.

Solution:
Vertical intercept : $f(0) = a = \dfrac{250}{\left(\frac{12}{25}\right)^{\frac{4}{3}}}$.
Practical interpretation: The amount of electric power in megawatts generated in the small country with coal produced in 2004.

b. [6 points] A small country decides to reduce the amount of electrical power they produce using coal. The electrical power W generated with coal in 2008 and 2011 was 250 and 120 megawatts, respectively.

i) Suppose that $W = g(t)$, where the function g is linear and t represents the number of years after 2004. Find a formula for $g(t)$. Show all your work.

Solution: The slope of $g(t)$ is $m = \frac{120-250}{7-4} = -\frac{130}{3}$. Using the point-slope formula we get $g(t) = 250 - \frac{130}{3}(t-4) = \frac{1270}{3} - \frac{130}{3}t$.

ii) Find the value of $g^{-1}(0)$. Include units. Show all your work.

Solution: Let $t = g^{-1}(0)$, then $\frac{1270}{3} - \frac{130}{3}t = 0$. Solving for $t = \frac{1270}{130} \approx 9.769$ years after 2004.