6. [14 points] After a day of work on the farm, Percy likes to toss corn cobs from the second story window of the barn to the ground. On one toss, the corn cob follows a parabolic path $h(x) = -x^2 + bx + c$ where $h(x)$ is the height of the cob above the ground, in feet, when it is a horizontal distance x feet from the barn. The numbers b and c are constants.

a. [3 points] Interpret the vertical intercept of $h(x)$ in the context of this problem.

b. [4 points] If the window is 9 feet from the ground, and the cob hits the ground 9 feet from the barn, find the values of the constants b and c. Show your work.

$$b = \underline{\hspace{3cm}}$$

$$c = \underline{\hspace{3cm}}$$

c. [4 points] After the cob bounces, it follows a path given by $p(x) = -\frac{1}{3}x^2 + 8x - 45$ where $p(x)$ is the height of the cob above the ground, in feet, when it is a horizontal distance x feet from the barn. By completing the square, find the maximum height the cob achieves after it bounces. You must show all steps of your calculation.

maximum height $= \underline{\hspace{3cm}}$

d. [3 points] Find the distance the cob is from the barn when it hits the ground for the second time. Show your work. Hint: Use the quadratic formula.

distance $= \underline{\hspace{3cm}}$