10. [8 points] The shape of Percy's favorite hill on his uncle's farm can be visualized as the graph of a piecewise function $y = f(x)$. The function is quadratic on the interval $[-5, 3)$, and it's exponential on the interval $[3, 10]$. The function satisfies the following properties:

- $x = -5$ is a zero of $f(x)$.
- $f(x)$ has y-intercept 10.
- $f(2) = 7$.
- $f(3) = 4$.
- For $3 \leq x \leq 9$, when x increases by one, $f(x)$ decreases by 20%.

Write a formula for $f(x)$. Your answer will be graded based on whether it satisfies the criteria in the problem.

$$f(x) = \begin{cases} -\frac{1}{2}(x+5)(x-4) & \text{for } -5 \leq x < 3 \\ 4(0.8)^{-3}(0.8)^x & \text{for } 3 \leq x \leq 10 \end{cases}$$

Solution: The quadratic part of $f(x)$ can be written $a(x+5)(x-r)$ since -5 is a zero. $f(0) = 10$, so $-5ar = 10$ or $-ar = 2$. We also know $f(2) = 7$, so $7 = 7a(2-r)$ or $1 = 2a - ar$. Combining these facts, we get $1 = 2a + 2$ or $a = -1/2$. This means $r = 4$.

The exponential part of $f(x)$ has growth factor 0.8 because it's decreasing by 20% for each increase in x by one, so we can write it as $a(0.8)^x$. Using $f(3) = 4$, we get $4 = a(0.8)^3$ or $a = 4(0.8)^{-3}$.