

2. [12 points] Consider the function  $y = p(x) = 2x^2 - \sqrt{33}x - 6$ .

- a. [4 points] Find the zeros of  $p(x)$  in exact form, if there are any, or explain why there aren't any. Show your work. Answers obtained using a calculator with no work shown will receive no credit.

The zeros of  $p(x)$  are  $\frac{\sqrt{33} \pm 9}{4}$

*Solution:* Using the quadratic formula, we have

$$x = \frac{\sqrt{33} \pm \sqrt{33 - 4(-6)(2)}}{4} = \frac{\sqrt{33} \pm 9}{4}$$

- b. [5 points] Find the  $x$ - and  $y$ -coordinates of the vertex of  $p(x)$  by completing the square. You must show all your steps and write  $p(x)$  in vertex form to receive credit.

The vertex of  $p(x)$  is  $(\frac{\sqrt{33}}{4}, -\frac{81}{8})$

*Solution:*  $p(x) = 2x^2 - \sqrt{33}x - 6$ .

$$p(x) = 2(x^2 - \frac{\sqrt{33}}{2}x) - 6.$$

$$p(x) = 2(x^2 - \frac{\sqrt{33}}{2}x + \frac{33}{16}) - 6 - 2(\frac{33}{16}).$$

$$p(x) = 2(x^2 - \frac{\sqrt{33}}{4})^2 - \frac{81}{8}.$$

- c. [3 points] Suppose  $p(x+h) = 2x^2 + \sqrt{33}x - 6$  for some number  $h$ . Find  $h$ . Support your answer with graphical or algebraic evidence.

$h = \frac{\sqrt{33}}{2}$

*Solution:* Completing the square for  $p(x+h)$  is identical to the calculation for  $p(x)$  except you have  $(x + \frac{\sqrt{33}}{4})^2$  instead of  $(x - \frac{\sqrt{33}}{4})^2$ . This means

$$p(x+h) = 2(x^2 + \frac{\sqrt{33}}{4})^2 - \frac{81}{8}.$$

This means the vertex of this new function is  $(-\frac{\sqrt{33}}{4}, -\frac{81}{8})$ , so the shift must have been  $\frac{\sqrt{33}}{2}$  to the left.