2. [12 points] Consider the function \( y = p(x) = 2x^2 - \sqrt{33}x - 6 \).

   a. [4 points] Find the zeros of \( p(x) \) in exact form, if there are any, or explain why there aren’t any. Show your work. Answers obtained using a calculator with no work shown will receive no credit.

   The zeros of \( p(x) \) are \( \pm \frac{\sqrt{33} + 9}{4} \).

   \[
   \text{Solution: Using the quadratic formula, we have}
   \]
   \[
   x = \frac{\sqrt{33} \pm \sqrt{33 - 4(-6)(2)}}{4} = \frac{\sqrt{33} \pm 9}{4}
   \]

   b. [5 points] Find the \( x \)- and \( y \)-coordinates of the vertex of \( p(x) \) by completing the square. You must show all your steps and write \( p(x) \) in vertex form to receive credit.

   The vertex of \( p(x) \) is \( \left( \frac{\sqrt{33}}{4}, -\frac{81}{8} \right) \).

   \[
   \text{Solution: } p(x) = 2x^2 - \sqrt{33}x - 6.
   \]
   \[
   p(x) = 2\left(x^2 - \frac{\sqrt{33}}{2}x\right) - 6.
   \]
   \[
   p(x) = 2\left(x^2 - \frac{\sqrt{33}}{2}x + \frac{33}{16}\right) - 6 - 2\left(\frac{33}{16}\right).
   \]
   \[
   p(x) = 2\left(x^2 - \frac{\sqrt{33}}{4}\right)^2 - \frac{81}{8}.
   \]

   c. [3 points] Suppose \( p(x + h) = 2x^2 + \sqrt{33}x - 6 \) for some number \( h \). Find \( h \). Support your answer with graphical or algebraic evidence.

   \[
   h = \frac{\sqrt{33}}{2}
   \]

   \[
   \text{Solution: Completing the square for } p(x + h) \text{ is identical to the calculation for } p(x) \text{ except you have } (x + \frac{\sqrt{33}}{4})^2 \text{ instead of } (x - \frac{\sqrt{33}}{4})^2. \text{ This means}
   \]
   \[
   p(x + h) = 2\left(x^2 + \frac{\sqrt{33}}{4}\right)^2 - \frac{81}{8},
   \]
   This means the vertex of this new function is \( \left( -\frac{\sqrt{33}}{4}, -\frac{81}{8} \right) \), so the shift must have been \( \frac{\sqrt{33}}{2} \) to the left.