4. [8 points] Hugo LeBlanc inadvertently introduced a rogue variety of algae in Lake Balkash at noon on June 12th. Kiki measured that the algae covered an area of 12.45 square meters on June 25th at noon, and she measured that the algae covered an area of 15.63 square meters on June 27th at noon.

a. [4 points] Assuming that the area covered by the algae grows exponentially, what was the initial area covered by the algae? Give your answer in exact form.

The algae initially covered $\underline{\quad 12.45/(\sqrt{\frac{15.63}{12.45}})^{13} \text{ square meters} \quad}$.

Solution: To find the initial amount a, we can write the equations

$$ab^{13} = 12.45$$
$$ab^{15} = 15.63$$

This means $b^2 = \frac{15.63}{12.45}$, so $b = \sqrt{\frac{15.63}{12.45}}$. We can now use either equation above to solve for a. For example, $a(\sqrt{\frac{15.63}{12.45}})^{13} = 12.45$. And then $a = 12.45/(\sqrt{\frac{15.63}{12.45}})^{13}$.

b. [4 points] Using exact form, find a formula for the area $A(t)$ that the rogue algae covers t days after Hugo introduced the algae. Be sure your formula is consistent with your answer from part (a).

$$A(t) = \underline{\quad \left(12.45/(\sqrt{\frac{15.63}{12.45}})^{13}\right)\left(\sqrt{\frac{15.63}{12.45}}\right)^t \quad}$$.

Solution: We did a lot of work in (a) already. We know

$$A(t) = ab^t = \left(12.45/(\sqrt{\frac{15.63}{12.45}})^{13}\right)\left(\sqrt{\frac{15.63}{12.45}}\right)^t$$