

5. [12 points] Jack is starting a business teaching others to paint. He has come up with the following pricing plan.
- For each lesson, a client has to pay a flat fee of \$6 to cover the cost of the art supplies they will use.
  - He charges \$2 per minute for the first 60 minutes of the lesson.
  - He charges \$0.50 per minute for each minute after that.
  - Each lesson lasts at most 120 minutes.

Let  $C(m)$  be the amount of money he charges for a lesson that is  $m$  minutes long.

- a. [2 points] Evaluate  $C(70)$ .
- b. [6 points] Find a formula for  $C(m)$ . Use standard piecewise function notation:

$$C(m) = \left\{ \right.$$

- c. [4 points] The function  $d = C(m)$ , where  $d$  is the cost (in dollars) of a painting lesson that lasts  $m$  minutes, is invertible. Write a formula for its inverse  $C^{-1}(d)$  using standard piecewise function notation.
6. [15 points] Scientists discover a new island in Lake Michigan and begin studying its animals. The island has both lizards and crows when they arrive, and they accidentally leave some mice on the island after discovering it.
- 5 thousand lizards live on the island when they discover it, but the population is decreasing at a rate of 5% per year.
  - Half a year after the island is discovered, the population of mice has grown to 2.3 times the initial population, and appears to be growing exponentially.
  - The population of crows, in thousands,  $t$  years after the island is discovered, can be modeled by  $C(t) = 4e^{0.06t-1}$ .

In the following problems, leave your answer in **exact form** and show every step of your work.

- a. [3 points] Find a formula for  $L(t)$ , the number of lizards on the island, in thousands,  $t$  years after the island is discovered.
- b. [3 points] How long does it take for the population of mice to reach 10 times the initial population?
- c. [2 points] What is the vertical intercept of  $C(t)$ ? Interpret the meaning of this number in the context of the problem.
- d. [2 points] By what percentage does the population of crows increase in a year?
- e. [5 points] When will there be the same number of lizards and crows on the island?