2. [8 points] Consider the following table of values for x, A, B, and C.

x	2	4	8	10
A	29	24	14	9
B	15.6	18.2	28.2	18.2
C	0	10	0	-3

For each of the following, decide whether the statement **could be true**. Briefly explain your reasoning.

a. B is a function of A.

Solution:

 True: Since the A-values in the table don't repeat, <u>any</u> of the other variables is a function of A. In particular, B is a function of A.

b. A is a function of C.

Solution: **False:** There is an input ($C = 0$) which corresponds to more than one output ($A = 29$ and $A = 14$), and so A <u>cannot</u> be a function of C.

c. A is a linear function of x.

Solution: **True:** As in part (a) above, since the A-values in the table don't repeat, A is a function of x. Moreover, note that the average rate of change $\frac{\Delta A}{\Delta x}$ is equal to $-\frac{5}{2}$ on all intervals in the table:

$$[2, 4]: \qquad \frac{\Delta A}{\Delta x} = \frac{24 - 29}{4 - 2} = -\frac{5}{2}$$

$$[4, 8]: \qquad \frac{\Delta A}{\Delta x} = \frac{14 - 24}{8 - 4} = -\frac{5}{2}$$

$$[8, 10]: \qquad \frac{\Delta A}{\Delta x} = \frac{9 - 14}{10 - 8} = -\frac{5}{2}$$

Therefore, A <u>could be</u> a linear function of x.

d. If $C = f(x)$, then $f(x)$ is concave down.

Solution:

 False: $f(x)$ has average rate of change

$$\frac{\Delta C}{\Delta x} = \frac{0 - 10}{8 - 4} = -\frac{5}{2}$$

on the interval $[4, 8]$, but $f(x)$ has average rate of change

$$\frac{\Delta C}{\Delta x} = \frac{-3 - 0}{10 - 8} = -\frac{3}{2}$$

on the interval $[8, 10]$. Since $-\frac{5}{2} < -\frac{3}{2}$, $\frac{\Delta C}{\Delta x}$ is <u>not</u> decreasing, and so $f(x)$ <u>cannot</u> be concave-down.