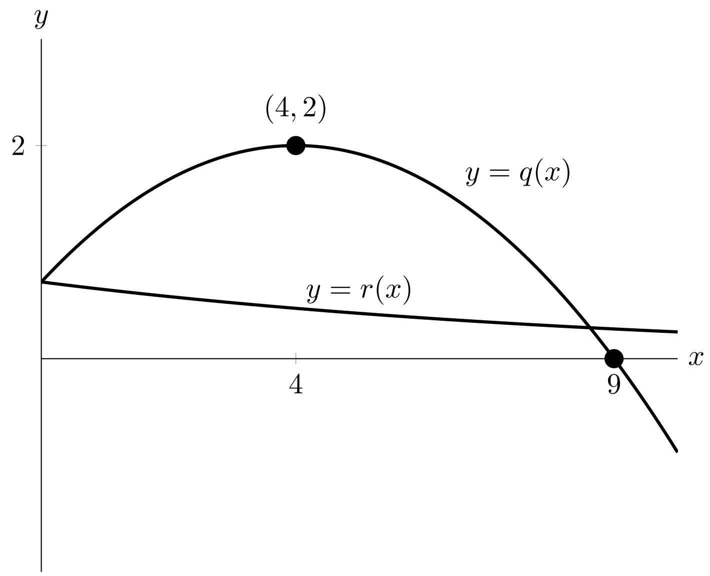
3. [11 points] The graph below shows **part** of

- a quadratic function $q(x)$ with vertex and one zero marked
- an exponential function $r(x) = ab^x$ that intersects $q(x)$ on the y-axis.



a. [4 points] Find a formula for $q(x)$.

Since $q(x)$ has vertex $(4, 2)$, it has a formula in vertex form of the form

$$q(x) = k(x-4)^2 + 2$$

for some non-zero real number k (which is the leading coefficient). We also know that 9 is a zero of $q(x)$; that is, $q(9) = 0$. We can use this equation to solve for k:

$$k(9-4)^2 + 2 = 0$$
$$k \cdot 25 = -2$$
$$k = -\frac{2}{25}$$

So $q(x)$ has formula $\boxed{q(x) = -\frac{2}{25}(x-4)^2 + 2}$.

b. [2 points] What is the x-coordinate of the other zero of $q(x)$?

Solution:
Recall that the vertex $(4, 2)$ lies on the vertical axis of symmetry $x = 4$ of the graph of $q(x)$. Since one zero $x = 9$ lies 5 units to the right of this axis, the other zero lies 5 units to the left of this axis. Thus $\boxed{x = -1}$ is the other zero of $q(x)$. We can confirm this by computing $q(-1)$ from our formula.
Alternatively, you can also solve the equation $q(x) = 0$, possibly by converting to a different form.

Recall that the formula for $r(x)$ is $r(x) = ab^x$. Use the graph and your formula for $q(x)$ to answer the following questions.

c. [3 points] Which of the options below **could** be true? Briefly explain your answer.

$a < 0$ $\qquad$ $0 < a < 1$ $\qquad$ $a > 1$

Solution:
Note that the graphs of $q(x)$ and $r(x)$ intersect along the y-axis. This implies that $q(0) = r(0)$. We can use this equation to solve for a:

$$a = ab^0 = r(0) = q(0) = -\frac{2}{25}(0-4)^2 + 2 = -\frac{32}{25} + \frac{50}{25} = \frac{18}{25}.$$

Thus $\boxed{0 < a < 1}$.

d. [2 points] Which of the options below **could** be true? Briefly explain your answer.

$b < 0$ $\qquad$ $0 < b < 1$ $\qquad$ $b > 1$

Solution:
Since $r(x)$ has positive initial value and is decreasing, it is exhibiting exponential decay, and so its growth factor b satisfies the inequality $\boxed{0 < b < 1}$.