4. [9 points] An ice cream shop along the Huron river in Ann Arbor is only open in the summer. Its owner has designed a model that predicts the revenue (that is, the amount of money the shop takes in) of the shop in thousands of dollars, P, on a day where the maximum temperature is T degrees Fahrenheit.The model is described by the function $P = g(T)$, and has an inverse, $g^{-1}(P)$.

The maximum temperature in Ann Arbor, in degrees Fahrenheit, on the d^{th} day that the shop is open for the summer, is given by the function $M(d)$.

For each of the following, either give a practical interpretation of the given expression, or explain why the expression doesn't make sense in the context of the problem.

a. [3 points] $g(M(13)) = 8$

Solution:

$M(13)$ is the maximum temperature (measured in degrees Fahrenheit) on the 13th day that the ice cream shop is open. $g(M(13))$ is the ice cream shop's revenue (measured in thousands of dollars) predicted by the model on that day. Therefore, the equation $g(M(13)) = 8$ has the following interpretation:

> The model predicts that the ice cream shop will take in \$8 thousand on the 13th day that it is open.

b. [3 points] $g^{-1}(5)$

Solution:

$g^{-1}(5)$ is the input to g whose output corresponds to 5. The function $P = g(T)$ takes as input a daily maximum temperature (measured in degrees Fahrenheit) and returns as output the revenue (measured in thousands of dollars) of the ice cream shop predicted by the model. Therefore, the expression $g^{-1}(5)$ has the following interpretation:

> the daily maximum temperature (measured in degrees Fahrenheit) at which the ice cream shop is predicted to take in \$5 thousand

c. [3 points] $M(g^{-1}(7))$

Solution:

For similar reasons as above, $g^{-1}(7)$ is a temperature measured in degrees Fahrenheit. Since the inputs to the function $M(d)$ are measured in days, not degrees Fahrenheit:

> It does not make sense to evaluate $M\left(g^{-1}(7)\right)$.