3. [8 points] Jaime is on a long car trip. Consider the following functions:

- Let $d(t)$ be the distance, in miles, Jaime has driven t minutes after they begin their trip.
- Let $g(t)$ be the amount of gas, in gallons, in Jaime's car's gas tank t minutes after they begin their trip.

Assume that both functions have inverses. For each part below, write a phrase or sentence giving a practical interpretation of the given expression or equation, or explain why it doesn't make sense in this context.

a. $d(9) = 4$

Solution: When Jaime has driven for 9 minutes, they've gone 4 miles.

b. $g(d^{-1}(120))$

Solution: the amount of gas, in gallons, in Jaime's car's tank when they've driven 120 miles

c. $g(60) = g(0) - 2$

Solution: 60 minutes into their trip, Jaime's car has 2 fewer gallons of gas than when their trip started.

4. [15 points] Mei is starting a coffee roasting business.

a. [4 points] Mei puts green coffee beans into her roaster. Let $T(t)$ be the temperature, in degrees Fahrenheit (°F), inside the roaster t minutes after she starts roasting the beans. Some values of $T(t)$ are given in the table below.

t	0	3	5	12
$T(t)$	70	370	470	320

Compute the average rate of change of $T(t)$ over the interval $[0, 5]$. **Include units.**

Solution: $\dfrac{470 - 70}{5 - 0} = \dfrac{400}{5} = 80$

Answer: **80°F per minute**

Could $T(t)$ be concave down on the entire interval $[0, 12]$? Show your work, and circle your final answer.

Solution: The average rates of change over the three consecutive subintervals are $\dfrac{370 - 70}{3 - 0} = 100$, $\dfrac{470 - 370}{5 - 3} = 50$, and $\dfrac{320 - 470}{12 - 5} < 0$. Since these are decreasing, yes, the function could be concave down on this interval.

Answer (*circle one*): **Yes** **No**

This problem continues onto the following page.

This problem continues from the previous page and is restated for your convenience.

Mei is starting a coffee roasting business.

b. [3 points] Let n be a variable representing the number of customers that come into her shop on the dth day it is open (so that $d = 1$ represents the first day she is open, etc.). Is it definitely true that d is a function of n? Briefly explain your answer.
Answer (*circle one*):

Yes, d must be a function of n | **No**, d might not be a function of n

Explanation:

Solution: It could be that on different days she is open, the same number of customers come into her shop, which would mean one input value (number of customers n) would have more than one output (day d she is open).

c. [5 points] Mei plans to sell her roasted coffee beans for \$15 per pound. However, she plans to offer a deal: once a customer has spent \$60 on coffee beans, any additional beans will only cost \$12 per pound. Find a piecewise-defined formula for $C(p)$, the cost to purchase p pounds of Mei's coffee beans.

Answer: $C(p) = \begin{cases} 15p & \text{for } 0 \le p \le 4 \\ 60 + 12(p-4) & \text{for } p > 4 \end{cases}$

d. [3 points] Compute $C^{-1}(75)$. Then, using a complete sentence and **including units**, give a practical interpretation of your answer in the context of the problem.

Solution: Because we need to find the value of p so that $C(p) = 75$, we know this will be for some $p > 4$, so we set the second piece of our function equal to 75. Solving, we find that

$$\begin{aligned} 75 &= 60 + 12(p-4) \\ \frac{15}{12} &= (p-4) \\ p &= \frac{5}{4} + 4 = \frac{21}{4} = 5.25 \end{aligned}$$

Answer: $C^{-1}(75) =$ **5.25**

Interpretation:

Solution: If a customer spent \$75 on coffee beans, they purchased 5.25 pounds of beans.