5. [8 points] For each part of this problem, you must **show every step** of any algebraic work that is required.

a. [3 points] The quadratic function $q(x)$ has its vertex at the point $(2, 4)$ and a zero at $x = 5$. Find a formula for $q(x)$.

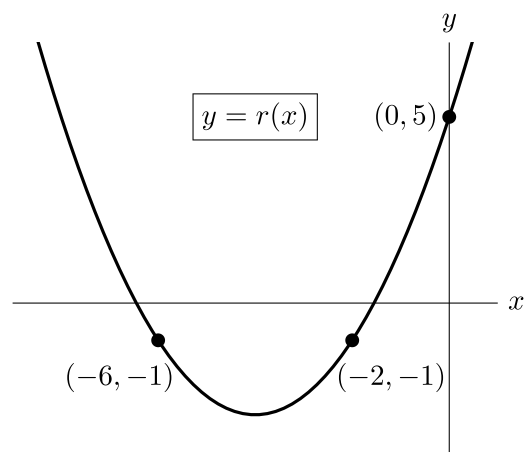
Solution: There are at least two possible solutions:

Using vertex form: we know that $q(x) = a(x-2)^2 + 4$ for some a. Then plugging in the point $(5, 0)$, we find $0 = a(5-2)^2 + 4 = 9a + 4$, so $a = \frac{-4}{9}$.

Using factored form: we know that, by symmetry, the function must have another zero at $x = -1$. Then we know that $q(x) = a(x-5)(x+1)$ for some a. Then plugging in the point $(2, 4)$, we find $4 = a(2-5)(2+1) = -9a$, so $a = \frac{-4}{9}$.

Answer: $q(x) =$ $\frac{4}{9}(x-2)^2 + 4$ **or** $\frac{4}{9}(x-5)(x+1)$

b. [5 points] Find a formula for the quadratic function $r(x)$ graphed below.



Solution:
There are at least three possible solutions:

If we first consider the function $p(x) = r(x) + 1$ by shifting the given graph up by 1, we can write

$$p(x) = a(x+6)(x+2).$$

Then noting that $(0, 6)$ is a point on $p(x)$, we have

$$6 = a(6)(2),$$

so that $a = \frac{1}{2}$. Then we know

$$r(x) = p(x) - 1 = \frac{1}{2}(x+6)(x+2) - 1.$$

We could also find a formula for $r(x)$ by writing $r(x) = a(x+4)^2 + k$, since we know by symmetry that the x-coordinate of the vertex is 4. Then we can plug in $(0, 5)$ and one of the other known points to solve a system of two equations for $a = \frac{1}{2}$ and $k = -3$. The formula obtained from this method is $r(x) = \frac{1}{2}(x+4)^2 - 3$.

Or, because we know the y-intercept is 5, we can write $r(x) = ax^2 + bx + 5$, and proceed similarly to the solution above using the other two points to solve for $a = \frac{1}{2}$ and $b = 4$. The formula obtained from this method is $r(x) = \frac{1}{2}x^2 + 4x + 5$.

Answer: $r(x) =$ $\frac{1}{2}(x+6)(x+2) - 1$