6. [17 points] A scientist is studying the mass, in milligrams (mg), of several different bacterial colonies.

- Colony A's mass is 17 mg at the start of the experiment, and it grows at a rate of 7% per hour.
- Colony B's mass in mg t hours after the experiment begins is given by $B(t) = 3e^{0.11t}$.
- Colony C's mass in mg t hours after the experiment begins is given by $C(t) = 22(1.04)^t$.
- Two hours into the experiment, Colony D has a mass of 21 mg, but by four hours into the experiment, only 18 mg remains.

For each part of this problem, you must **show every step** of any algebraic work that is required.

a. [3 points] Find a formula for the function $A(t)$, which gives the mass, in mg, of colony A t hours after the experiment begins.

Answer: $A(t) =$ $17(1.07)^t$

b. [2 points] By what percent is colony B growing each hour? Give your answer in exact form or rounded to at least two decimal places.

Answer: $100(e^{0.11} - 1) \approx 11.63$ %

c. [3 points] How many hours will it take for colony B's population to triple? *Give your answer in exact form, and circle your final answer.*

Solution:

$$
\begin{aligned}
3e^{0.11t} &= 9 \\
e^{0.11t} &= 3 \\
0.11t &= \ln(3) \\
t &= \frac{\ln(3)}{0.11}
\end{aligned}
$$

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This problem continues from the previous page and is restated for your convenience.

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d. [5 points] At what time t will the size of colonies B and C be the same?
Give your answer in exact form, and circle your final answer.

Solution: There are at least two solutions:

We can take the natural log of both sides first:

$$\begin{aligned} 3e^{0.11t} &= 22(1.04)^t \\ \ln(3) + \ln(e^{0.11t}) &= \ln(22) + \ln(1.04)^t \\ \ln(3) + 0.11t &= \ln(22) + t\ln(1.04) \\ 0.11t - t\ln(1.04) &= \ln(22) - \ln(3) \\ t(0.11 - \ln(1.04)) &= \ln(22) - \ln(3) \\ t &= \frac{\ln(22) - \ln(3)}{0.11 - \ln(1.04)} \end{aligned}$$

We can also rearrange the equation first:

$$\begin{aligned} 3e^{0.11t} &= 22(1.04)^t \\ \frac{e^{0.11t}}{(1.04)^t} &= \frac{22}{3} \\ \left(\frac{e^{0.11}}{1.04}\right)^t &= \frac{22}{3} \\ t\ln\left(\frac{e^{0.11}}{1.04}\right) &= \ln\left(\frac{22}{3}\right) \end{aligned}$$

$$t = \frac{\ln\left(\frac{22}{3}\right)}{\ln\left(\frac{e^{0.11}}{1.04}\right)}$$

e. [4 points] Assuming colony D's mass is decaying exponentially, what will its mass (in mg) be 12 hours after the start of the experiment? *Give your answer in exact form.*

Solution: We can plug in the points $(2, 21)$ and $(4, 18)$ into $y = ab^t$, giving $21 = ab^2$ and $18 = ab^4$. Dividing, this leads to $\frac{18}{21} = b^2$, or $b = \left(\frac{6}{7}\right)^{1/2}$.

Then we can solve for a by plugging b into, say, $21 = ab^2$, which leads to $a = \frac{21}{\frac{6}{7}} = \frac{21 \cdot 7}{6} = 24.5$.
Finally, we can find the mass when $t = 12$ by using the values we found for a and b along with $t = 12$:

$$24.5\left(\frac{6}{7}\right)^6$$

Alternately, once we have b, we can instead note that the mass after 12 hours will be 18, the mass at 4 hours, times b^8.

Answer: $24.5\left(\frac{6}{7}\right)^6$