3. [11 points] The UM Dance Club met up with the UM Math Modeling Club to write formulas for different dancer's jumps. They measure one dancer's total time in the air as 1 second and their maximum height as 4 feet. They know that the function $D(t)$ which gives the dancer's height (in feet) as a function of time after they jump (in seconds) is a quadratic function.

a. [3 points] One member of the Math Modeling Club wants to find the formula for $D(t)$ using the zeros of the function, so is starting with the form:

$$D(t) = a(t-r)(t-s)$$

To model the dancer's jump described above, what are possible values of r and s and how do you know?

$r =$ **0**

$s =$ **1**

Explanation:

Solution: The zeros of this function are when the dancer's height is 0. If the dancer's total time in the air is 1 second, that means that they are on the ground (height 0) at $t = 0$ seconds and then again 1 second later at $t = 1$ second. Since these are the zeros of $D(t)$, they are the values of r and s in the factored form taken as the starting point.

b. [3 points] Another member of the Math Modeling Club wants to write a formula using vertex form of a quadratic function:

$$D(t) = a(t-h)^2 + k$$

To model the dancer's jump described above, what are the values of h and k in this formula and how do you know?

$h =$ **0.5**

$k =$ **4**

Explanation:

Solution: $D(t)$ is given in vertex form this time, so (h, k) is the coordinates of the vertex. Because the zeros are at $t = 0, 1$, the t-value of the vertex must be halway between them, at $t = 0.5$. This is why $h = 0.5$. We're also told that the dancer's maximum height is 4ft, so this gives us the vertical coordinate of the vertex, and $k = 4$.

The UM Dance Club met up with the UM Math Modeling Club to write formulas for different dancer's jumps. They measure one dancer's total time in the air as 1 second and their maximum height as 4 feet. They know that the function $D(t)$ which gives the dancer's height (in feet) as a function of time after they jump (in seconds) is a quadratic function.

c. [3 points] Find the exact value of a in the formulas above. *You can use either of your equations to do this. Show all work.*

Solution: We can solve this using either starting point: factored form OR vertex form. For the sake of the solutions, we show both ways. (This is also a way to verify that our work is correct!). For both methods, we'll plug in an additional known point into our starting equations, and then solve for the value of a.

Method 1: plug in vertex $(0.5, 4)$ into factored form from part (a) and solve for a.

$$\begin{aligned} 4 &= a(0.5-0)(0.5-1) = a(0.5)(-0.5) \\ 4 &= a(-0.25) \\ 4/-0.25 &= a \\ -16 &= a \end{aligned}$$

Method 2: plug in either zero ($(0,0)$ or $(1,0)$) into vertex form from part (b) and solve for a.

$$\begin{aligned} 0 &= a(0-0.5)^2 + 4 \\ -4 &= a(-.5)^2 \\ -4 &= a(0.25) \\ -4/0.25 &= a \\ -16 &= a \end{aligned}$$

$a =$ **-16**

d. [2 points] From the context of the problem alone—without relying on or referring to your calculation above—would you expect the value of a to be positive or negative? Why?

$a > 0$ $\boxed{a < 0}$ NOT ENOUGH INFORMATION

Explanation:

Solution: Because this dancer reaches a maximum height, this must be a "downward facing" or concave-down parabola. For such a parabola, the leading coefficient, a, must be negative.