4. [11 points] The UM Youtubers Club makes a very cool new video that goes viral. Suppose the video had 100 views at 2:00AM Eastern Time (ET) Saturday morning and the number of views grew exponentially for at least the next 24 hours, with views doubling every hour.

a. [1 point] Between 3:00AM ET and 6:00AM ET Saturday, by what factor had the number of views increased?

Solution: Because the number of views doubles each hours, over three hours it will double three times. In other words, it will increase by a factor of $2^3 = 8$.

Factor of increase: $2^3 = 8$

b. [2 points] Write a formula for a function $V = f(t)$, where V is the number of video views and t is the number of hours since 2:00AM ET Saturday.

Solution: We know the number of views at 2:00AM is 100, and that it doubles each hour after that. That means the growth factor is $b = 2$. Putting this together we get the exponential function $V = f(t) = 100 \cdot 2^t$.

$V = f(t) =$ $100 \cdot 2^t$

c. [6 points] For each of the following expressions or equations, explain its meaning in the context of the problem, or explain why it doesn't make sense in the context of the problem.

(i) $f^{-1}(500,000) \approx 12.25$

Solution: The time at which the number of views reaches 500,000 is approximately 12.25 hours after 2AM, or 2:15PM.

(ii) $\frac{f(5)-f(3)}{5-3} = 1200$

Solution: Between 5AM and 7AM, the views increased, on average, by 1200 views per hour.

d. [2 points] Write a new function $g(s)$ **in terms of** f that would give us the number of views the video had s hours after 9:00AM ET on Saturday morning.

Solution: Graphical perspective: if our new starting point is 9:00AM, this is like shifting our graph so what was previously at $t = 7$ is now at $s = 0$. So this is, graphically speaking, a shift left by 7. This means our new formula is $g(s) = f(s + 7)$.

Points perspective: If I put $s = 0$ (9AM) into our new function g, this should give the same output as $t = 7$ (9AM) into our original function $f(t)$. This examples shows that we need to add 7 to our s values before putting them into f. This yields the same result as above: $g(s) = f(s + 7)$.

$g(s) =$ $f(s + 7)$