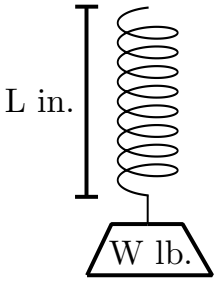
5. [7 points] The UM Weights and Measures Club is building a spring scale, which weighs objects by hanging them from a spring.

Let W be the weight of an object, in pounds, and let L be the length of the spring in inches when we hang that object from it. It turns out that there is a linear relationship between W and L. The club observes that their spring is 3 inches long when no weight is attached, and stretches out to 5.5 inches when they test it with a 5-pound weight.



a. [3 points] What is the slope of the function $W = f(L)$? Explain the meaning of the slope's value in the context of the problem.

Solution: The slope is "change in input divided by change in output." In this case, we're told that the weight (output) changes by 5 lbs and the length of the spring (input) changes by $5.5 - 3 = 2.5$ inches. This means our slope is $5/2.5 = 2$ pounds per inch.

Another perspective: We know two points on the graph of this function: $(3, 0)$ and $(5.5, 5)$. Using those two poitns we can find the slope of the line between them and arrive at the same answer as above.

Slope = **2**

Meaning:

Solution: Our slope's units of "pounds per inch" (coming from "change in output / change in input") are useful here! "2 pounds per inch" mean that for each additional inch the spring stretches, there had been 2 more pounds added to the scale.

b. [2 points] Find a formula for $W = f(L)$.

Solution: Because we know the slope is 2 and we know that $(3, 0)$ is on the graph of the function, we can use point-slope form to find our equation:

$$W = 2(L - 3) + 0 = 2(L - 3) = 2L - 6$$

We get the same formula if we use the other known point $(5.5, 5)$:

$$W = 2(L - 5.5) + 5 = 2L - 11 + 5 = 2L - 6$$

$W =$ $2L - 6$

c. [2 points] Suppose we hang a bucket from the spring and then pour in some water. As we add the weight of the water, the spring gets 4 inches longer. How much does the added water weigh? *Include units.*

Solution: Since we know the slope is 2 lbs / inch, this means that for each inch longer the spring grows, there was a corresponding addition of 2 lbs of weight. So if the spring got 4 inches longer, that came from added weight of $4 \times 2 = 8$ pounds.

Note that is doesn't work to plug 4 inches into our formula for W because the problem is not saying the spring was 4 inches total, but saying that it *lengthened* by 4 inches. That is, the change is four inches, but not the total.

The water in the bucket weighs **8 pounds**