7. [8 points] The following table shows some values of 3 different functions:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
<th>$h(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>27</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>25</td>
<td>24</td>
</tr>
</tbody>
</table>

This blank space is for your work and calculations.

**Solution:**
Examine function $f$:
- Average rate of change of $f$ over $[6, 7]$: $-9$; average rate of change of $f$ over $[7, 9]$: $-10/2 = -5$. These are not the same, so $f$ cannot be linear.
- $f$ is decreasing
- These average rates of change are increasing as we go left to right, so $f$ could be concave up.
- From $x = 6$ to $x = 7$, $f(x)$ changes by a factor of $18/27 = 2/3$. To see if that same growth factor is in play from $x = 7$ to $x = 9$, we need to apply it twice.

$$
\frac{2}{3} \cdot \frac{2}{3} \cdot f(7) = \frac{2}{3} \cdot \frac{2}{3} \cdot 18 = \frac{2}{3} \cdot 12 = 8 = f(9)
$$

This means the same growth factor holds from $x = 7$ to $x = 9$, so $f$ could be exponential with growth factor of $2/3$.

Examine function $g$:
- Average rate of change of $g$ over $[6, 7]$: 5; average rate of change of $g$ over $[7, 9]$: $5/2 = 5/2$. These are not the same, so $g$ cannot be linear.
- $g$ is increasing
- These average rates of change are decreasing as we go left to right, so $f$ could be concave down (but not concave up).
- From $x = 6$ to $x = 7$, $g(x)$ changes by a factor of $20/15 = 4/3$. To see if that same growth factor is in play from $x = 7$ to $x = 9$, we need to apply it twice.

$$
\frac{4}{3} \cdot \frac{4}{3} \cdot g(7) = \frac{4}{3} \cdot \frac{4}{3} \cdot 20 = \frac{4}{3} \cdot \frac{80}{3} = \frac{320}{3} \neq g(9) = 25
$$

This means that the $4/3$ growth factor does not continue to hold, so $g(x)$ cannot be exponential.

Examine function $h$:
- Average rate of change of $h$ over $[6, 7]$: 4; average rate of change of $h$ over $[7, 9]$: $8/2 = 4$. These are the same, so $h$ could be linear.
- $h$ has a positive slope and is increasing
- Since the average rates of change are constant, they are neither increasing or decreasing, so $h(x)$ can be neither concave down nor concave up.
- Since $h(x)$ has a constant rate of change, it cannot be exponential (since all exponential functions have changing rates of change).

Circle all correct options for each part.
a. [2 points] Which of these functions could be linear?

\[
\begin{array}{ccc}
  f(x) & g(x) & h(x) \\
  \text{NONE OF THESE}
\end{array}
\]

b. [2 points] Which of these functions could be exponential?

\[
\begin{array}{ccc}
  f(x) & g(x) & h(x) \\
  \text{NONE OF THESE}
\end{array}
\]

c. [2 points] Which of these functions could be concave up on the interval \([6, 9]?)

\[
\begin{array}{ccc}
  f(x) & g(x) & h(x) \\
  \text{NONE OF THESE}
\end{array}
\]

d. [2 points] Which of these functions could be increasing on the interval \([6, 9]?)

\[
\begin{array}{ccc}
  f(x) & g(x) & h(x) \\
  \text{NONE OF THESE}
\end{array}
\]