7. [8 points] The following table shows some values of 3 different functions:

x	6	7	9
$f(x)$	27	18	8
$g(x)$	15	20	25
$h(x)$	12	16	24

This blank space is for your work and calculations.

Solution:

Examine function f:

- Average rate of change of f over $[6,7]$: -9; average rate of change of f over $[7,9]$: $-10/2 = -5$. These are not the same, so f cannot be linear.
- f is decreasing
- These average rates of change are increasing as we go left to right, so f could be concave up.
- From $x = 6$ to $x = 7$, $f(x)$ changes by a factor of $18/27 = 2/3$. To see if that same growth factor is in play from $x = 7$ to $x = 9$, we need to apply it *twice*.

$$\frac{2}{3} \cdot \frac{2}{3} \cdot f(7) = \frac{2}{3} \cdot \frac{2}{3} \cdot 18 = \frac{2}{3} \cdot 12 = 8 = f(9)$$

  This means the same growth factor holds from $x = 7$ to $x = 9$, so f could be exponential with growth factor of $2/3$.

Examine function g:

- Average rate of change of g over $[6,7]$: 5; average rate of change of g over $[7,9]$: $5/2 = 5/2$. These are not the same, so g cannot be linear.
- g is increasing
- These average rates of change are decreasing as we go left to right, so f could be concave down (but not concave up).
- From $x = 6$ to $x = 7$, $g(x)$ changes by a factor of $20/15 = 4/3$. To see if that same growth factor is in play from $x = 7$ to $x = 9$, we need to apply it *twice*.

$$\frac{4}{3} \cdot \frac{4}{3} \cdot g(7) = \frac{4}{3} \cdot \frac{4}{3} \cdot 20 = \frac{4}{3} \cdot \frac{80}{3} = \frac{320}{3} \neq g(9) = 25$$

  This means that the $\frac{4}{3}$ growth factor does not continue to hold, so $g(x)$ cannot be exponential.

Examine function h:

- Average rate of change of h over $[6,7]$: 4; average rate of change of h over $[7,9]$: $8/2 = 4$. These are the same, so h could be linear.
- h has a positive slope and is increasing
- Since the average rates of change are *constant*, they are neither increasing or decreasing, so $h(x)$ can be neither concave down nor concave up.
- Since $h(x)$ has a constant rate of change, it cannot be exponential (since all exponential functions have changing rates of change).

Circle **all** correct options for each part.

a. [2 points] Which of these functions could be linear?

$f(x)$ $g(x)$ $\boxed{h(x)}$ NONE OF THESE

b. [2 points] Which of these functions could be exponential?

$\boxed{f(x)}$ $g(x)$ $h(x)$ NONE OF THESE

c. [2 points] Which of these functions could be concave up on the interval $[6, 9]$?

$\boxed{f(x)}$ $g(x)$ $h(x)$ NONE OF THESE

d. [2 points] Which of these functions could be increasing on the interval $[6, 9]$?

$f(x)$ $\boxed{g(x)}$ $\boxed{h(x)}$ NONE OF THESE