2. [9 points] The height of water in a cylindrical tank, as it drains out, is given by

$$H = h(t) = 4t^2 - 40t + 100,$$

where H is measured in centimeters and t is measured in minutes after a spigot is opened. The formula holds until the tank is emptied, after which, the height does not change anymore.

For your reference, the zeros of $y = ax^2 + bx + c$ *can be found by the formula:* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a. [2 points] How high is the water in the tank when the spigot is first opened? *Give your answer in exact form, or rounded to two decimal places. Include units.*

Solution: $h(0) = 4(0)^2 - 40(0) + 100 = 100$

Water height: 100 **centimeters**

b. [2 points] After how many minutes is the tank empty? *Show all work. Give your final answer in exact form or rounded to two decimals places.*

Solution: We need to find the first t-value for which $0 = 4t^2 - 40t + 100$.
We can first try factoring to solve:

$$\begin{aligned} 0 &= 4t^2 - 40t + 100 \\ 0 &= 4(t^2 - 10t + 25) \\ 0 &= 4(t-5)(t-5) \end{aligned}$$

Since this was factorable, we can see that the only solution is when $t = 5$ minutes.

5 minutes

c. [2 points] What is a reasonable domain and range for this function in the context of the problem? *Use inequality OR interval notation for your answer.*

Domain: $[0, 5]$ **Range:** $[0, 100]$

d. [3 points] How long does it take for the tank to be half as full as it started? *Show all work. Give your final answer rounded to two decimals places.*

Solution: We need to find out how long it takes for the water to reach 50cm. That is, we need to solve the following equation for t:

$$\begin{aligned} 50 &= 4t^2 - 40t + 100 \\ 0 &= 4t^2 - 40t + 50 \\ 0 &= 2t^2 - 20t + 25 \end{aligned}$$

We can use the quadratic formula to find all possible solutions:

$$\begin{aligned} t &= \frac{20 \pm \sqrt{400 - 200}}{4} \\ t &= \frac{20 \pm \sqrt{200}}{4} \\ t &= \frac{20 \pm 10\sqrt{2}}{4} \\ t &= 5 \pm \frac{5\sqrt{2}}{2} \end{aligned}$$

Only one of these solutions actually occurs before the tank is empty (during the range of this problem), and that is the smaller one. So our only solutions is $t = 5 - \frac{5\sqrt{2}}{2} \approx 1.464466$

$5 - \frac{5\sqrt{2}}{2} \approx 1.464466$ minutes