5. [6 points] Let $W(d)$ be the probability that the great soccer player Pelénomial scores when he takes a shot d yards away from the goal line. Some values of $W(d)$ are given in the table below.

d	0	6	12	18
$W(d)$	0.94	0.4831	0.2483	

a. [2 points] Is $W(d)$ modeled better by a linear function or by an exponential function? *To receive credit, you must test* both *models* and *show all work.*

Solution: To check if the function is linear, we can calculate average rates of change:

$$\frac{0.4831 - 0.94}{6} = -0.07615$$

$$\frac{0.2483 - 0.4831}{6} = -0.0391\overline{3}$$

Since these average rates of change are not equal (or even close to equal), the function $W(d)$ does not seem to be well modeled by a linear function.
To check if $W(d)$ is approximately exponential, we can check if successive ratios outputs (from evenly spaced inputs) are equal (or close to equal). Since our inputs are evenly spaced, we need only check the following ratios:

$$\frac{0.2483}{0.4831} \approx 0.514$$

$$\frac{0.4831}{0.94} \approx 0.514$$

Since they are approximately equal (and the slopes calculated above are not at all close), we can conclude that the exponential function in a better fit.

(Circle one) LINEAR $\boxed{\text{EXPONENTIAL}}$

b. [2 points] If you said above that $W(d)$ was linear, find its slope. If you said above that $W(d)$ was exponential, find its approximate growth factor. *Show all work or point to relevant work above. Give your answer rounded to two decimal places.*

Solution: Let us call the growth factor b. Then from the above table and work we know that: $b^6 \approx 0.514$. So $b \approx 0.514^{\frac{1}{6}} \approx 0.895$

SLOPE (if linear) / GROWTH FACTOR (if exponential): 0.895

c. [2 points] Use your work above to compute the probability that Pelénomial scores when he takes a shot 18 yards away from the goal line. *Show all work. Give your answer in exact form, or rounded to two decimal places.*

Solution: One way to compute this is to notice that for every 6 yds, we found above that the probability is multiplied by 0.514. So the probability at 18 yards should be 0.514 times the probability at 12 yards, or $0.2483 \times 0.514 \approx 0.1276$.
We could also approach this by using the fact that our growth factor above tells us that for each additional yard, the probability is multiplied by 0.895. So to find the probability at 18 yards, we could start with the probability at 0 yards and get: $0.94 \times (0.895)^{18} \approx 0.1276$

Answer: ≈ 0.1276