**3**. [11 points] The cost and amount of memory in computers has changed dramatically over time. For the $t^{th}$ year after 2000,

- let $C(t)$ be the cost, in dollars, of 1 gigabyte (GB) of memory in year $t$, and
- let $M(t)$ be the average amount of memory, in GB, in a new computer in year $t$.

**a**. [2 points] The function $C(t)$ is decreasing over its domain $t \geq 0$. Briefly explain why this means that $C$ must be invertible, that is, why the inverse $C^{-1}$ must also be a function.

*Solution:* Since $C$ is decreasing, as we move to the right the output values get smaller, so no two output values come from the same input. For $C^{-1}$, then, no input will have more than one associated output, as needed.

Alternately, since $C$ is decreasing, as time went on the cost of memory decreased, so no year had the same cost. This means we can also view the year as a function of the cost, and this is the function $C^{-1}$.

**b**. [2 points] Suppose that the average rate of change of $C(t)$ over the interval $[10, 16]$ was $-1.3$. Interpret what this means in the context of the problem.

*Solution:* Between the years 2010 and 2016, the cost of 1 GB of memory decreased by \$1.30 each year on average.

**c**. [5 points] Assume that $M$ and $C$ are both invertible functions. Describe the meaning of each of the following expressions or equations in the context of this problem, or explain why the expression or equation doesn't make sense in context.

i. $M(15) = 8$

*Solution:* In 2015, the average amount of memory in a new computer was 8 GB.

ii. $M^{-1}(4)$

*Solution:* the number of years after 2000 when the average amount of memory in a new computer was 4 GB

iii. $M(C(10))$

*Solution:* This does not make sense in context, because $C$ outputs a cost in dollars, while $M$ expects a number of years as its input.

**d**. [2 points] Write down a mathematical equation that represents the following sentence.

*When the average amount of memory in a new computer was $\frac{1}{2}$ GB, the average cost of each GB was \$100.*

**Answer:** $C\left(M^{-1}\left(\frac{1}{2}\right)\right) = 100$

Note: there are other equivalent answers.