6. [11 points] Jose's business is now selling fuzzy gloves. He has 100 pairs to sell for \$6 a pair. Jose's friend Neil has his own business making fuzzy fabric and makes a deal with Jose: if Jose sells all 100 pairs of gloves, Neil will provide fabric that will allow Jose to sell up to 300 additional pairs for \$9 a pair, as long as Neil gets \$2 from each pair sold.

Jose wants a function $J(p)$ for the amount of money, in dollars, he would make if he sold p pairs of gloves and gave Neil his share, if applicable.

a. [1 point] What is the domain of J in the context of this problem? Use either inequality or interval notation.

Domain: $[0, 400]$

b. [4 points] Write a piecewise-defined formula for the function $J(p)$ on its domain.

Answer: $J(p) = \begin{cases} 6p & \text{for } 0 \le p \le 100 \\ 600 + 7(p - 100) & \text{for } 100 \le p \le 400 \end{cases}$

Neil also wants a function $N(p)$ for the amount of money, in dollars, he would make if Jose sells p pairs of gloves.

c. [3 points] Write a piecewise-defined formula for the function $N(p)$.

Answer: $N(p) = \begin{cases} 0 & \text{for } 0 \le p \le 100 \\ 2(p - 100) & \text{for } 100 \le p \le 400 \end{cases}$

d. [3 points] If the fabric Neil plans to give Jose costs \$500, how many pairs of gloves, in total, does Jose have to sell for Neil to recoup his costs, that is, for Neil to make \$500? Show all of your work.

Solution: We set $2(p - 100) = 500$ and solve to find that $p = 350$.

Answer: 350